Control Systems

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Third Year ECE

Unit-I

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Lecture 8

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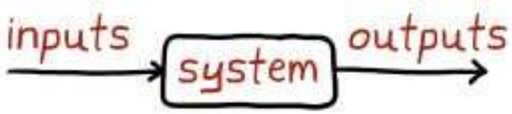
UNIT- I

- Introduction to Control system
  - Control System – Definition and Practical Examples
  - Basic Components of a Control System
- Feedback Control Systems:
  - Feedback and its Effect
  - Types of Feedback Control Systems
- Block Diagrams:
  - Representation and reduction
  - Signal Flow Graphs
- Modeling of Physical Systems:
  - Electrical Networks and Mechanical Systems
  - Force-Voltage Analogy
  - Force-Current Analogy

Mathematic Modeling of Dynamical Systems



- The set of mathematical equation describing the dynamic characteristics of a system is called mathematical model of a system.
- Dynamics/ mathematical equation of many systems can be written in terms of differential equations
  - Mechanical, thermal, electrical, economic, biological systems etc.
- We said that these D.E.'s can be derived using basic physical laws
- All systems we will study will be 'causal', i.e. the system's response at any time 't' depends only on past and not future inputs
- Recall transfer functions:
  - It is the ratio of Laplace Transform of output to Laplace Transform of input, when initial conditions are zero.
  - We assume
    - Zero initial conditions
    - Linearity
    - Time Invariance

Similarities in Mechanical and Electrical Systems

3 basic components in mechanical systems:

- Mass
- Spring
- Damper

✓ 3 basic components in electrical systems:

- Resistance
- Capacitor
- Inductor

- Basic form of differential equations is the same.
- Therefore, learning to model one type of system easily leads to modeling method for the other.
- Also, electrical and mechanical systems can be easily cascaded in block diagrams due to this similarity.
- In fact, many other types of systems have similar forms
- We will begin with electrical systems. This will make modeling mech easier!

Modelling Electrical Systems (Nise)

- Current (i) is the rate of flow of charge (q)

$$i(t) = \frac{dq(t)}{dt} \quad \Rightarrow \quad q(t) = \int i(t)\,dt$$

- Taking Laplace transform

$$I(s) = sQ(s) \quad \Rightarrow \quad Q(S) = \frac{1}{s} I(s)$$

- Impedance (complex resistance) is defined as

$$z = \frac{v}{i} \quad \Rightarrow \quad Z(s) = \frac{V(s))}{I(s)}$$

- It's mathematical inverse is called admittance.

Passive Electric Components

- Resistor:
  - $v(t) = i(t)R = R\frac{dq(t)}{dt} \Rightarrow V(s) = I(s)R$
  - $i(t) = \frac{v(t)}{R} \Rightarrow I(s) = \frac{V(s)}{R}$
- Capacitor:
  - $v(t) = \frac{1}{C}q(t) = \frac{1}{C}\int i(t)dt \Rightarrow V(s) = \frac{1}{Cs}I(s)$
  - $i(t) = C\frac{dv(t)}{dt} \Rightarrow I(s) = CsV(s)$
- Inductor:
  - $v(t) = L\frac{di(t)}{dt} = L\frac{d^2q(t)}{dt^2} \Rightarrow V(s) = LsI(s)$
  - $i(t) = \frac{1}{L}\int v(t)dt \Rightarrow I(s) = \frac{1}{Ls}V(s)$

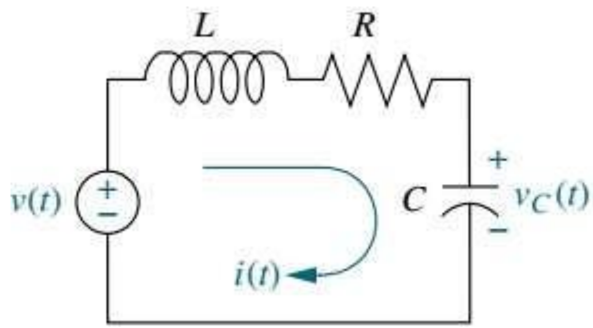
- We will combine these elements in complex networks using Kirchoff's Laws
  - – Current and Voltages in a loop sum to zeros

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^1 i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^1 v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), C – F (farads), R – Ω (ohms), G – Ω (mhos), L – H (henries).

Single Loop RLC Circuit

- Find transfer function of $V_c(s)$ to input V(s)



Summing the voltages around the loop, assuming zero initial conditions, yields the integro-differential equation for this network as

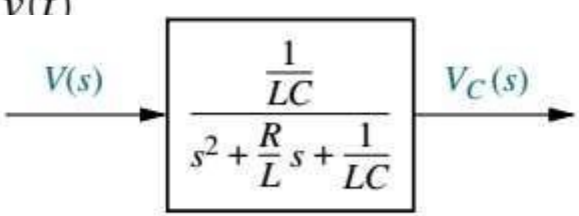
$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = v(t)$$

Changing variables from current to charge using $i(t) = dq(t)/dt$ yields

$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t) = v(t)$$

From the voltage-charge relationship for a capacitor

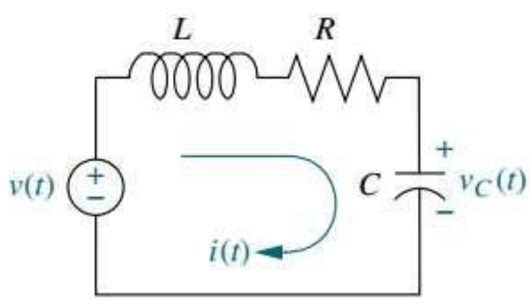
$$q(t) = Cv_C(t)$$

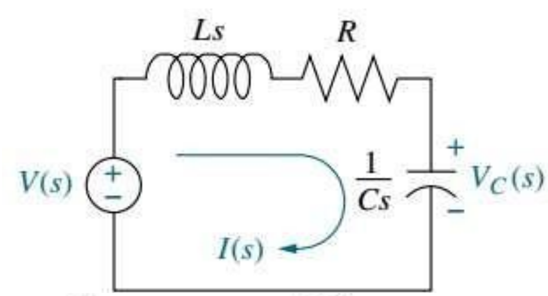


$$LC\frac{d^2v_C(t)}{dt^2} + RC\frac{dv_C(t)}{dt} + v_C(t) = v(t) \;\Rightarrow\; (LCs^2 + RCs + 1)V_C(s) = V(s)$$

Simplifying the Procedure

- Let us look at this in another way.
  - Resistor: $V_R(s) = I(s)R$
  - Capacitor: $V_C(s) = \frac{1}{Cs} I(s)$
  - Inductor: $V_L(s) = LsI(s)$
- Let's define impedance (similar to resistance) as $Z(s) = \frac{V(s)}{I(s)}$
- Unlike resistance, impedance is also applicable to capacitors & inductors.
- It represents information about dynamic behavior of components.





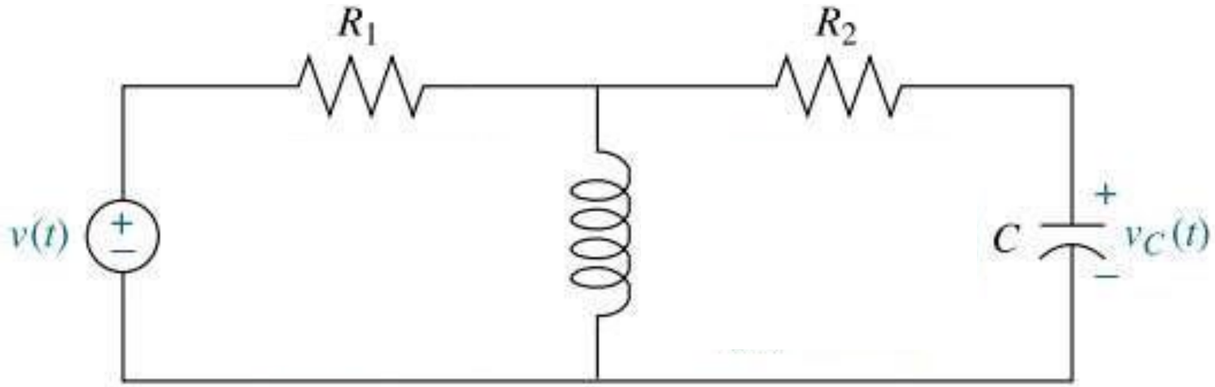
$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = v(t)$$

$$\left(Ls + R + \frac{1}{Cs}\right)I(s) = V(s)$$

[Sum of impedances]$I(s)$ = [Sum of applied voltages]

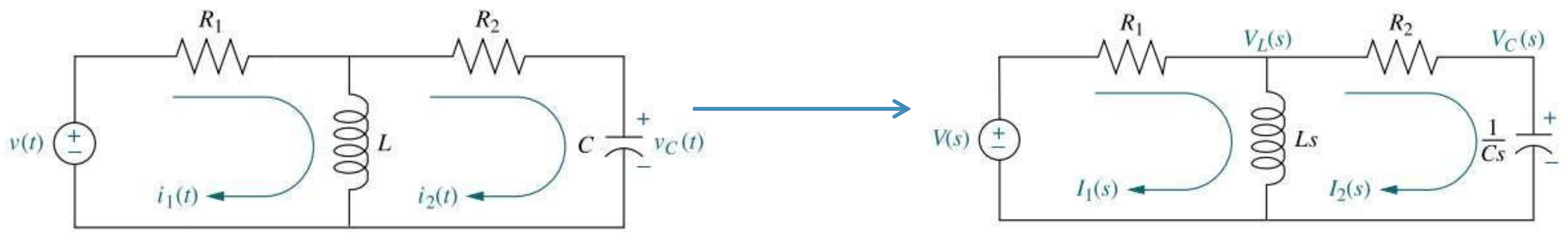
Solving Multi-Loop Electric Circuits

- For multiple loops and loads, use the following recipe.
  - Replace passive element values with their impedances.
  - Replace all sources and time variables with their Laplace transform.
  - Assume a transform current and a current direction in each mesh.
  - Write Kirchhoff's voltage law around each mesh.
  - Solve the simultaneous equations for the output.
  - Form the transfer function.



Multi-loop Example

- Find the transfer function $I_2(s)$ / V (s)



- Solving for Loop 1 and Loop 2

$$(R_1 + Ls)I_1(s) \qquad - LsI_2(s) = V(s) \qquad (1)$$

$$- LsI_1(s) + \left(Ls + R_2 + \frac{1}{Cs}\right)I_2(s) = 0 \qquad (2)$$

- There are various ways to solve this.

$$\begin{bmatrix} R_1 + Ls & -Ls \\ -Ls & Ls + R_2 + 1/Cs \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V(s) \\ 0 \end{bmatrix}$$

- This will yield the following transfer function

$$G(s) = \frac{I_2(s)}{V(s)} = \frac{Ls}{\Delta} = \frac{LCs^2}{(R_1 + R_2)LCs^2 + (R_1R_2C + L)s + R_1}$$

Summarizing the Method

- Let us look at the pattern in the last example

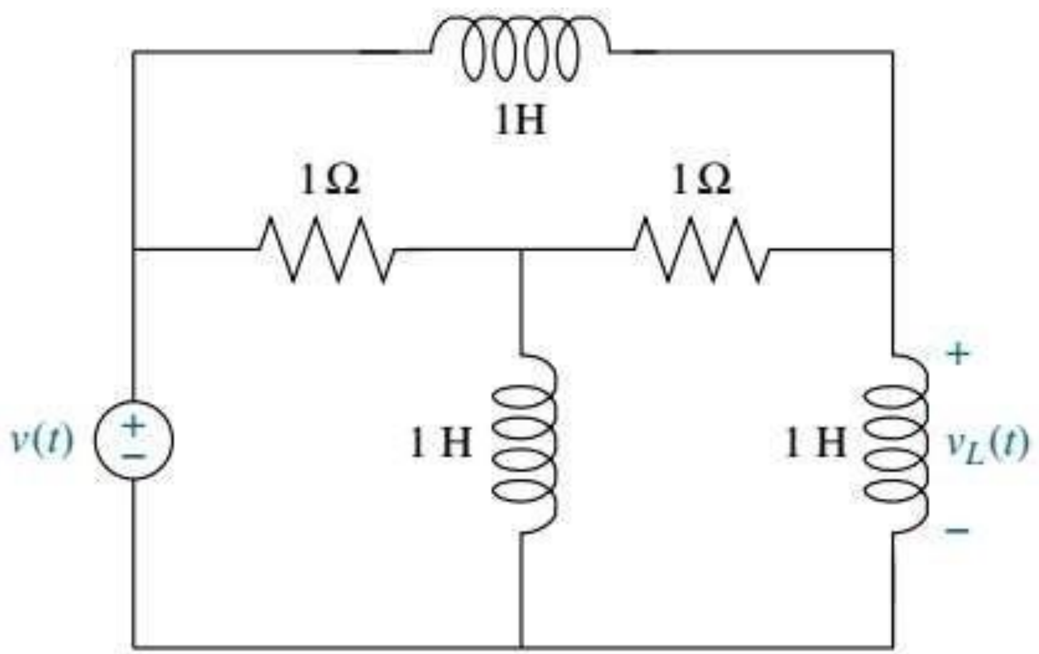
$$\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{around Mesh 1}\end{bmatrix} I_1(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to the}\\ \text{two meshes}\end{bmatrix} I_2(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{voltages around}\\ \text{Mesh 1}\end{bmatrix}$$

$$-\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{common to the}\\ \text{two meshes}\end{bmatrix} I_1(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{around Mesh 2}\end{bmatrix} I_2(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{voltages around}\\ \text{Mesh 2}\end{bmatrix}$$

- This form will help us write such equations rapidly
- Mechanical equations of motion (covered next) have the same form. So, this form is very useful!

Class Quiz

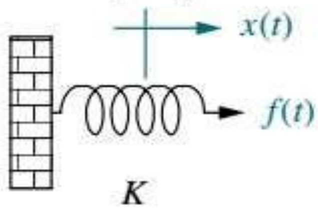
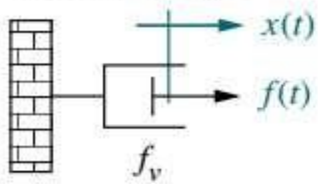
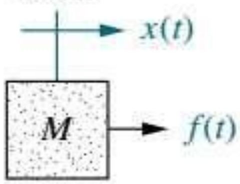
PROBLEM: Find the transfer function, $G(s) = V_L(s)/V(s)$, for the circuit given



ANSWER: $V_L(s)/V(s) = (s^2 + 2s + 1)/(s^2 + 5s + 2)$

Mechanical Systems (Translational)

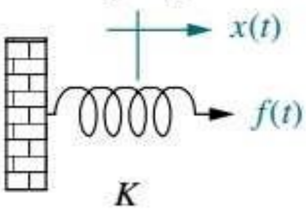
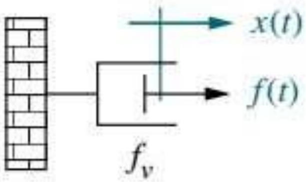
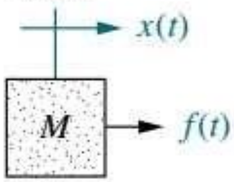
- Many concepts applied to electrical networks can also be applied to mechanical systems via analogies.
- This will also allow us to model hydraulic/pneumatic/thermal systems.

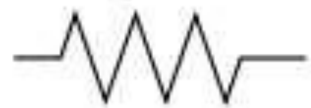
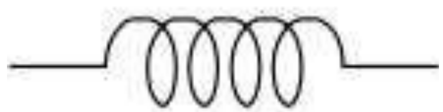
Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Electrical/Mechanical Analogies

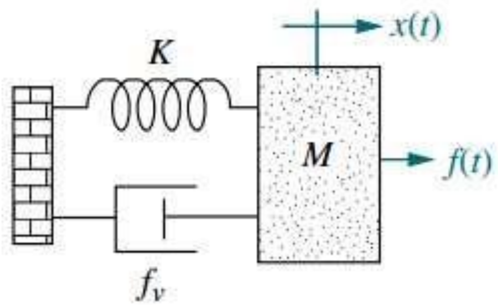
- Mechanical systems, like electrical networks, have 3 passive, linear components:
  - Two of them (spring and mass) are energy-storage elements; one of them, the viscous damper, dissipates energy.
  - The two energy-storage elements are analogous to the two electrical energy-storage elements, the inductor and capacitor. The energy dissipater is analogous to electrical resistance.
- Displacement ‘x’ is analogous to current I
- Force ‘f’ is analogous to voltage ‘v’
- Impedance (Z=V/I) is therefore Z=F/X
- Since, [Sum of Impedances] I(s) = [Sum of applied voltages]
- Hence, [Sum of Impedances] X(s) = [Sum of applied forces]

Electric Mechanical Analogy

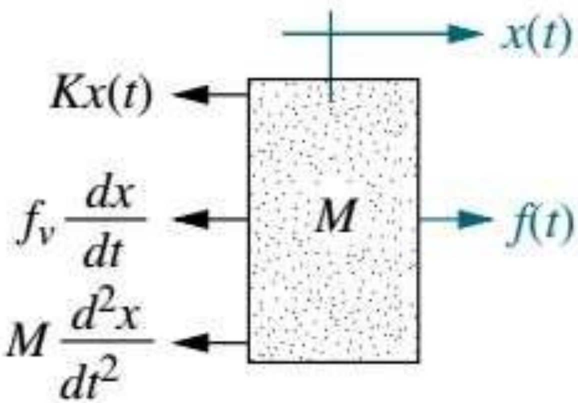
Component	Impedence $Z_M(s) = F(s)/X(s)$
Spring 	K
Viscous damper 	$f_v s$
Mass 	Ms^2

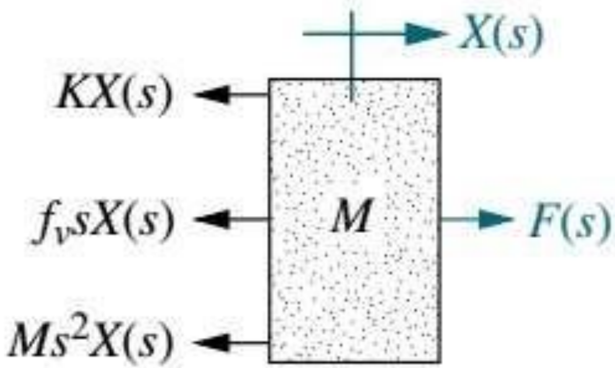
Component	Impedance $Z(s) = V(s)/I(s)$
 Capacitor	$\frac{1}{Cs}$
 Resistor	R
 Inductor	Ls

Spring Mass Damper System



Find the transfer function, $X(s)/F(s)$





$$M\frac{d^2x(t)}{dt^2}+f_v\frac{dx(t)}{dt}+Kx(t)=f(t)$$

$$(Ms^2+f_v s+K)X(s)=F(s)$$

Solving for the transfer function yields

$$G(s)=\frac{X(s)}{F(s)}=\frac{1}{Ms^2+f_v s+K}$$

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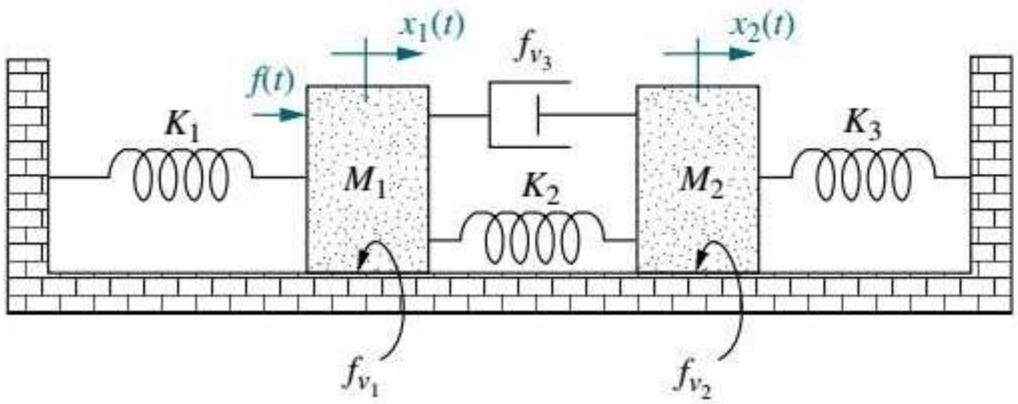
Transfer Function??

Find the transfer function, $X_2(s)/F(s)$



- System has two degrees of freedom, since each mass can be moved in the horizontal direction while the other is held still.
- 2 simultaneous equations of motion will be required to describe system.
- The two equations come from free-body diagrams of each mass.
- Forces on M_1 are due to (a) its own motion and (b) motion of M_2 transmitted to M_1 through the system.