Unit-2

Network Analysis & Synthesis

SYLLABUS

- Review of Laplace transforms
- Poles and zeroes,
- Initial and final value theorems,
- Transform circuit,
- Thevenin’s and Norton’s theorems,
- System function,
- Step and impulse responses,
- Convolution integral.
- Amplitude and phase responses.
- Network functions,
- Relation between port parameters,
- Transfer functions using two port parameters,
- Interconnection of two ports

Review of Laplace Transforms

- L.T. of signal f(t) is defined is as:

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt \qquad (1)$$

where $s = \sigma + j\omega, \sigma$ is decaying factor, ω is angular frequency.

- Put the value of s in above equation:

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-\sigma t} e^{-j\omega t} dt \qquad (2)$$

  - Hence for F(s) to be convergent, $\left| \int_{-\infty}^{\infty} f(t) e^{-\sigma t} dt \right|$ should be finite
  - Based on the given f(t), there will be σ values for which the above condition is satisfied **i.e. L.T. is defined**.
  - Range of σ values are called as **Region of Convergence (ROC)** of L.T. of F(s)
  - It is represented either in terms of σ or Re{s}

- Two varieties of LT:
  - **Unilateral or one-sided ($0<t<\infty$)**
  - **Bilateral or two-sided ($-\infty<t<\infty$)**
- B.L.T. of some basic signals
  - $f(t)=\delta(t)$

    $\left| \int_{-\infty}^{\infty} f(t)e^{-\sigma t}dt \right| < \infty$ for all values of σ

    $F(s)= \int_{-\infty}^{\infty} f(t)e^{-st}dt=1$ with entire ROC as entire s plane

    $$\mathbf{L}[\delta(t)] \quad \Leftrightarrow \quad \mathbf{1 \text{ for all } Re\{s\}}$$

  - $f(t)=u(t)$

    $\left| \int_{-\infty}^{\infty} f(t)e^{-\sigma t}dt \right| < \infty$ for $\sigma > 0$

    $F(s)= \int_{-\infty}^{\infty} f(t)e^{-st}dt=\frac{1}{s}$ with $Re\{s\}>0$

    $$\mathbf{L}[u(t)] \quad \Leftrightarrow \quad \frac{\mathbf{1}}{\boldsymbol{s}} \mathbf{\text{ for all } Re\{s\}>0}$$

Similarly,

$$\mathbf{L[-u(}t\mathbf{)]} \quad \Leftrightarrow \quad -\frac{1}{s} \text{ for all Re\{s\}<0}$$

$$\mathbf{L[-u(-}t\mathbf{)]} \quad \Leftrightarrow \quad \frac{1}{s} \text{ for all Re\{s\}<0}$$

$$\mathbf{L[tu(}t\mathbf{)]} \quad \Leftrightarrow \quad \frac{1}{s^2} \text{ for all Re\{s\}>0}$$ Ramp signal

$$\mathbf{L[}t^n\mathbf{u(}t\mathbf{)]} \quad \Leftrightarrow \quad \frac{n!}{s^{n+1}} \text{ for all Re\{s\}>0}$$

- f’(t)=$e^{-at}f(t)$

$$\int_{-\infty}^{\infty} e^{-at}f(t)e^{-st}dt = \int_{-\infty}^{\infty} f(t)e^{-(s+a)t}dt = F(s+a)$$

e.g.

$e^{-at}f(t) \longleftrightarrow \frac{1}{s+a}$ Re{s+a}>0 or σ>-a

$e^{at}f(t) \longleftrightarrow \frac{1}{s-a}$ Re{s-a}>0 or σ>a

Poles and Zeroes:

- The Laplace transform is rational, i.e. it is a ratio of polynomials in the complex variable '*s*'

$$\mathbf{F}(\boldsymbol{s}) = \frac{\boldsymbol{N}(\boldsymbol{s})}{\boldsymbol{D}(\boldsymbol{s})}$$

where *N* and *D* are the numerator and denominator polynomial, respectively.

- The roots of *N*(*s*) are known as the **zeros**. For these values of *s*, *F*(*s*) is zero.
- The roots of *D*(*s*) are known as the **poles**. For these values of *s*, *F*(*s*) is infinite.
- The set of poles and zeros completely characterise *F*(*s*) to within a scale factor (+ ROC for Laplace transform)

$$\mathbf{F}(\boldsymbol{s}) \propto \frac{\prod_i (\boldsymbol{s}-\boldsymbol{z}_i)}{\prod_j (\boldsymbol{s}-\boldsymbol{p}_j)}$$

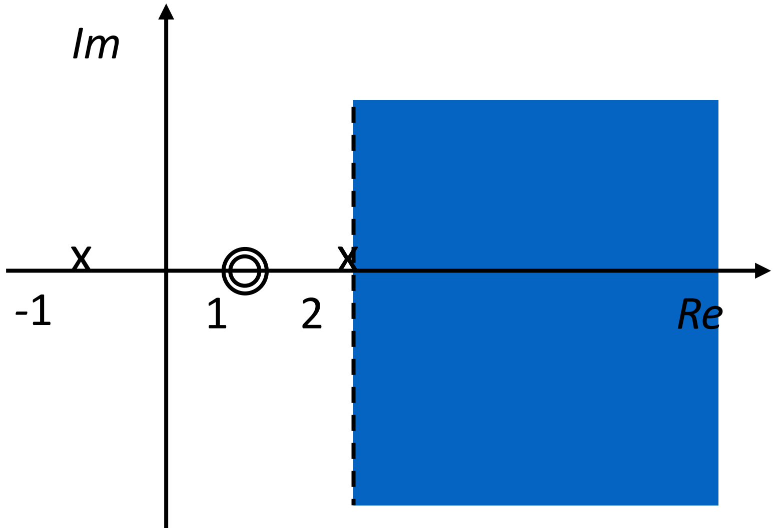
Continued…

- For e.g.

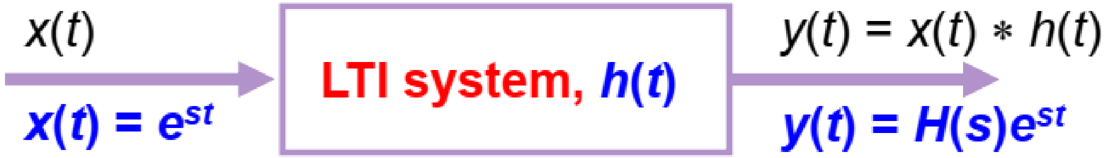
$$F(s) = 1 - \frac{4}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s-2} = \frac{(s-1)^2}{(s+1)(s-2)}, \qquad \mathrm{Re}\{s\} > 2$$

contains 2 poles at s=1 and 2 zeros at s=-1,-2

- S-plane



The Transfer Function



- Recall that the transfer function $H(s)$ of an LTI system is $H(s)=\int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau$
- If we take the bilateral Laplace transform of y(t), then

$$Y(s)=H(s)X(s) \Rightarrow H(s)=\frac{Y(s)}{X(s)}$$

This definition applies only at values of s for which $X(s) \neq 0$

- From differential equation: $\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t)$

After Substituting e^{st} for $x(t)$ and $e^{st}H(s)$ for $y(t)$, we obtain rational transfer function

$$\left(\sum_{k=0}^{N} a_k \frac{d^k}{dt^k}\{e^{st}\}\right)H(s) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k}\{e^{st}\} \Rightarrow H(s)=\frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} = \frac{\tilde{b}\prod_{k=1}^{M}(s-c_k)}{\prod_{k=1}^{N}(s-d_k)}$$

- Knowledge of the poles d_k, zeros c_k, and factor $\tilde{b} \equiv b_M / a_N$ completely determine the system

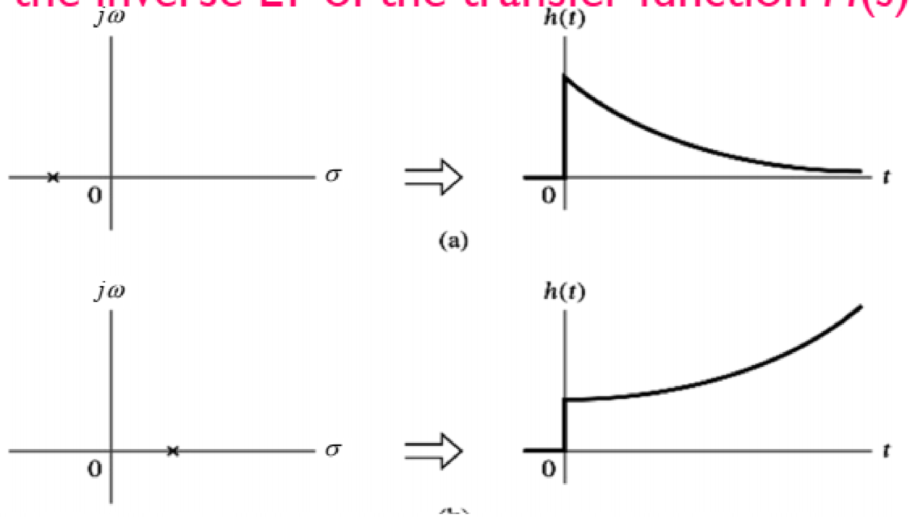
Properties of ROC of L.T.:

- The ROC of F(s) consist of strips parallel to jω axis in s-plane
- The ROC of F(s) does not contain any poles, although it is always bounded by poles.
- If f(t) is absolutely integrable and of finite duration, then the ROC is the entire s-plane
- If f(t) is right sided sequence, then the ROC extends outward from the outermost pole in F(s)
- If f(t) is left sided sequence, then the ROC extends inward from the innermost pole in F(s)
- LTI system with h(t) to be stable, the ROC must include $\sigma = 0$ (jω axis)
- LTI system with h(t) to be causal, the ROC must be right sided.
- LTI system with h(t) to be stable and causal, the ROC must be right sided and including $\sigma = 0$ (jω axis) line.

Causality and Stability

- The impulse response $h(t)$ is the inverse LT of the transfer function $H(s)$
- Causality

  right-sided inverse LT



- Stability

  the ROC includes the $j\omega$-axis in the s-plane