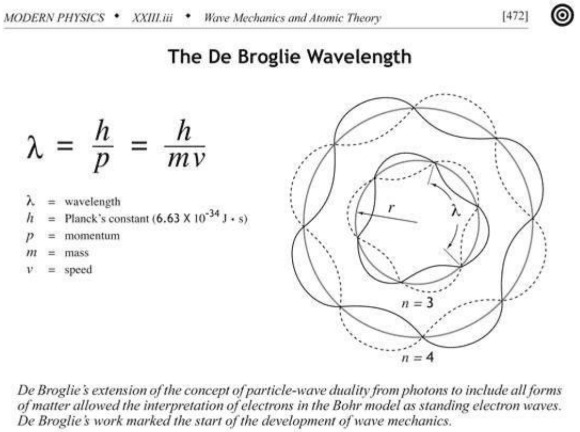
QUANTUM MECHANICS

UNIT II
Quantum Mechanics
Lecture-1

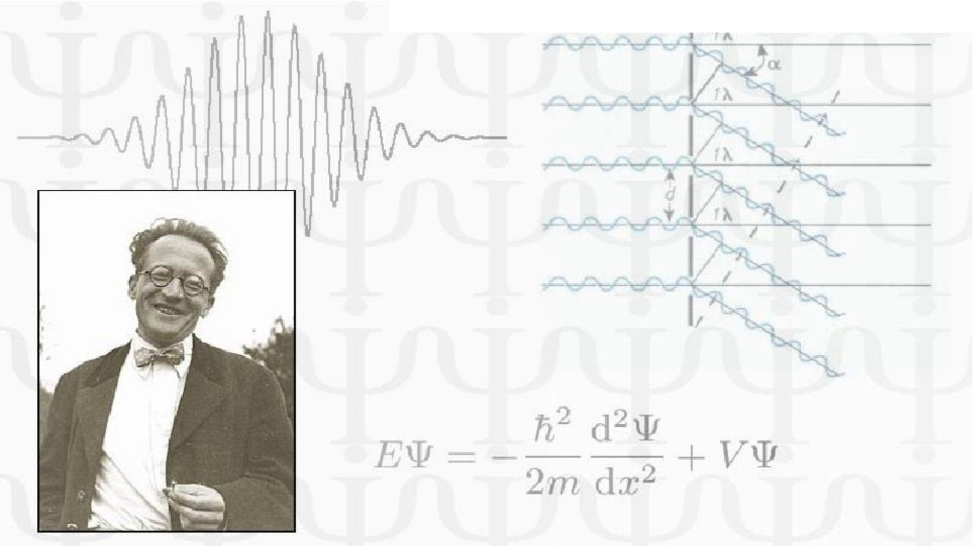

MODERN PHYSICS • XXIII.iii • Wave Mechanics and Atomic Theory [472]

The De Broglie Wavelength

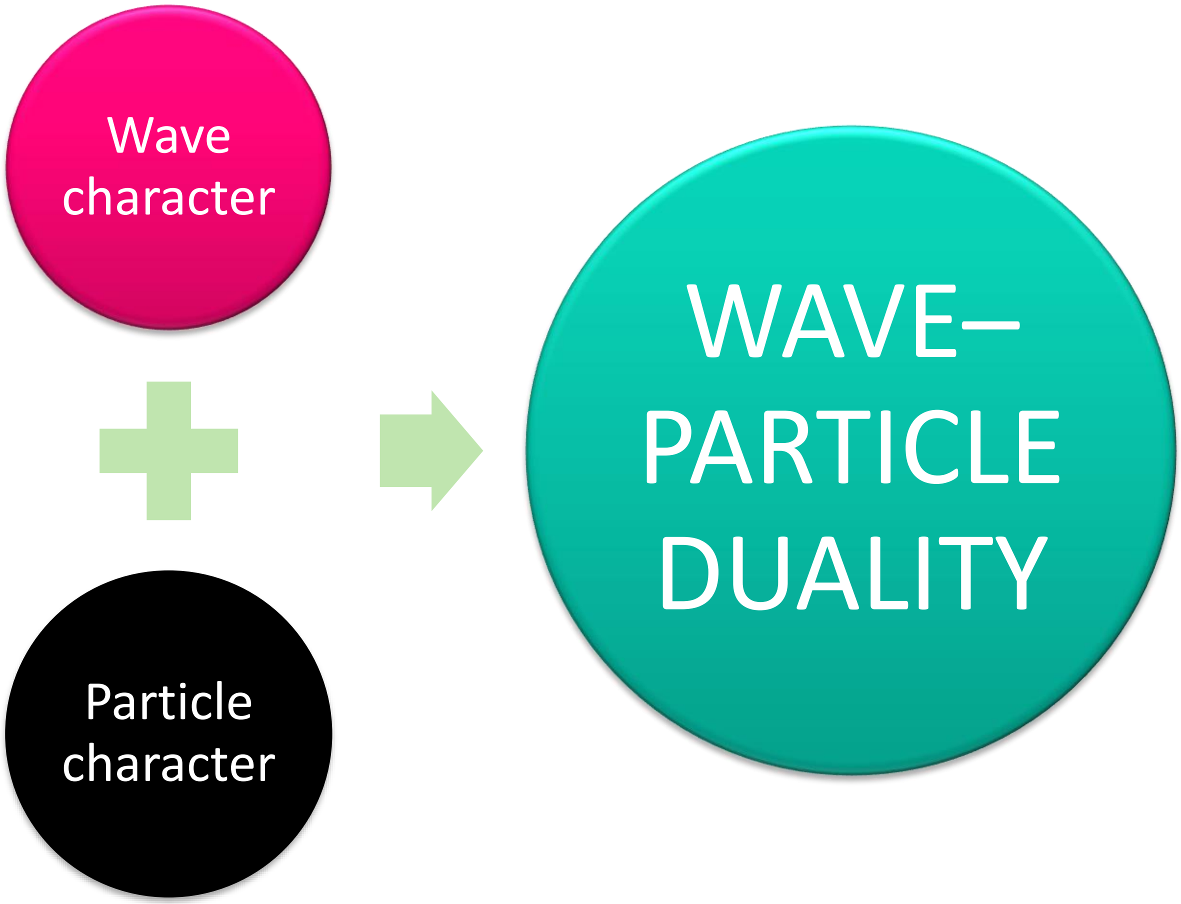
$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

λ = wavelength
h = Planck's constant (6.63×10^{-34} J • s)
p = momentum
m = mass
v = speed

De Broglie's extension of the concept of particle-wave duality from photons to include all forms of matter allowed the interpretation of electrons in the Bohr model as standing electron waves. De Broglie's work marked the start of the development of wave mechanics.



Broglie Matter-Waves and Wave–Particle Duality



Nature of a wave

- A wave is described by frequency (ν), wavelength (λ), phase velocity (u) and intensity (I)

- A wave is spread out and occupies a relatively large region of space

Nature of a particle

- A particle is specified by mass (m), velocity (v), momentum (p), and energy (E).

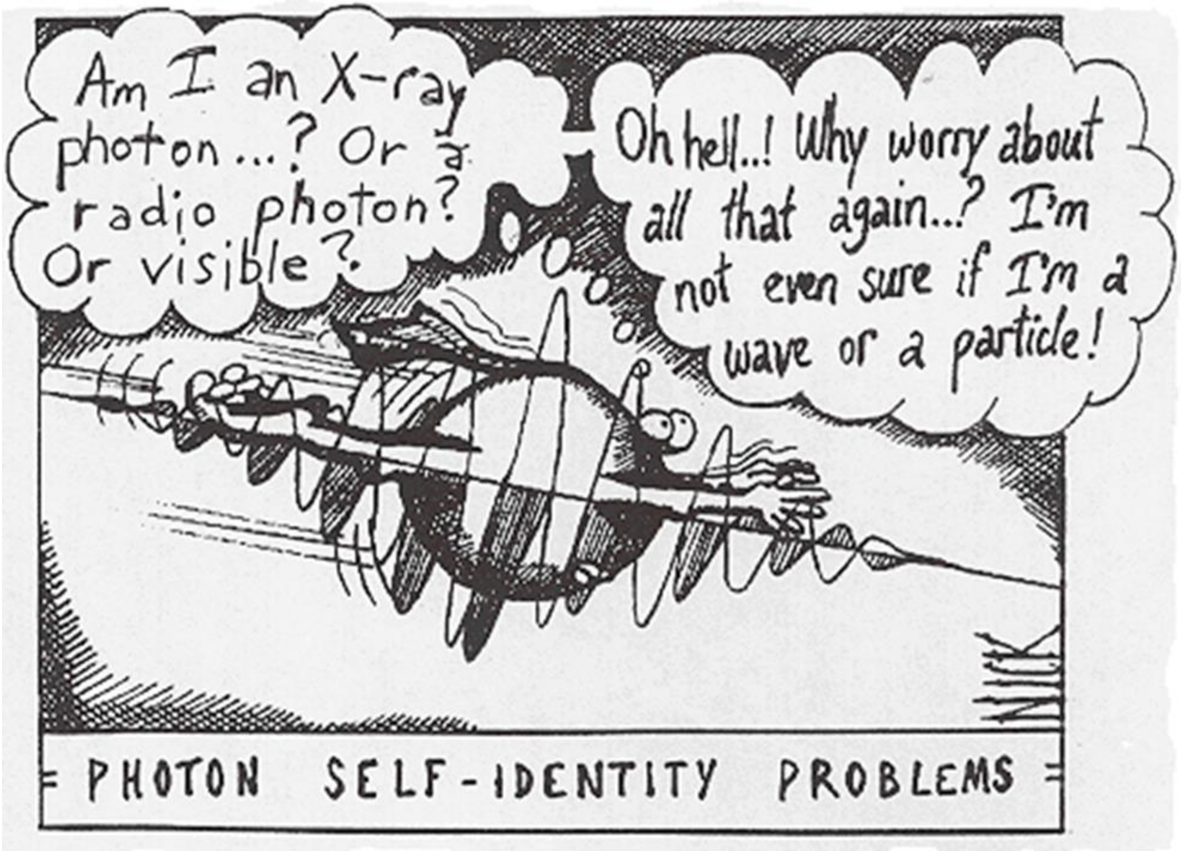
- A particle occupies a definite position in space.

Light

- Interference and Diffraction experiments showed the wave nature of light

- Blackbody radiation and Photoelectric effect can be explained only by considering light as a stream of particles

So, Is light a wave or a particle ?



WAVE–PARTICLE DUALITY

concept of a particle

- Mass: A particle must have definite mass.
- Velocity: A particle can move from one place to another with a certain velocity.
- Position: A particle may be located at some definite place or point.
- Momentum: A particle having mass and velocity possesses momentum during its motion.
- Energy: A particle has energy in different forms in different situations such as potential energy, kinetic energy, rest-mass energy.

WAVE–PARTICLE DUALITY

concept of a wave

- Lack of position: A wave is always realised as a disturbance; it is spread out over a relatively large region of space. It cannot be located at some definite place or point.
- Mass: It is very difficult to think about the mass being associated with a wave.
- Frequency and wavelength: A wave or disturbance, which advances in a medium, has a certain frequency and wavelength.
- Phase of wave velocity: Phase gives an idea about the instantaneous position and direction of a wave.
- Amplitude: The amplitude of a wave gives an idea of the intensity of disturbance in the medium.

DE BROGLIE HYPOTHESIS

In the Year 1924 Louis de Broglie made the bold suggestion

" If radiation which is basically a wave can exhibit particle nature under certain circumstances, and since nature loves symmetry, then entities which exhibit particle nature ordinarily, should also exhibit wave nature under suitable circumstances"

The reasoning used might be paraphrased as follows

1. **Nature loves symmetry**

2. **Therefore the two great entities, matter and energy, must be mutually symmetrical**

3. **If energy (radiant) is undulatory and/or corpuscular, matter must be corpuscular and/or undulatory**

DE BROGLIE HYPOTHESIS CONTD…..

- If light can act like a wave sometimes and like a particle at other times, then *all matter*, usually thought of as particles, should exhibit wave-like behaviour

- The relation between the momentum and the wavelength of a photon can be applied to material particles also

De- Broglie Wavelength

wavelength

$$\lambda = \frac{h}{p}$$

momentum

Relates a particle-like property (p)
to a wave-like property (l)

DE BROGLIE WAVELENGTH

- The Wave associated with the matter particle is called Matter Wave.
- The Wavelength associated is called de Broglie Wavelength.

➢ deBrogliewavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$, h is Planck's Constant, m is the mass of the particle, v is the velocity of the particle

➢ For an electron with Kinetic Energy E accelerated by a potential difference V

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$$

➢ substituting the required values we get

$$\lambda = \frac{6.625\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.602\times10^{-19}\times V}} = \frac{1.226}{\sqrt{V}}\,\text{nm}$$

➢ Thus for $V = 100\text{Volts}$

$$\lambda = \frac{1.226}{\sqrt{100}} = 0.1226\text{nm}$$

The Frequency

- De Broglie postulated that all particles satisfy Einstein's relation

- $E = h\nu$

- In other words,

$$\nu = \frac{E}{h}$$

de BROGLIE HYPOTHESIS OF MATTER-WAVES

- According to this theory, light may be considered as a stream of photons (particles) having mass ($h\nu/c^2$), energy ($h\nu$), velocity (c), and momentum ($h\nu$ /c).
- However, this theory (photon/Quantum theory) could not explain the phenomena like interference, diffraction, and polarisation.
- It indicates that the various phenomena of light (radiation) can be made only on the basis of the dual nature of light (radiation).
- Thus, light has dual nature, i.e., it possesses both particle and wave nature. It is important to remember that wave and particle nature can never appear together.

de BROGLIE HYPOTHESIS OF MATTER-WAVES

- Extending the idea of wave–particle duality of radiation (light), Louis de Broglie in 1924 suggested that this duality is true not only for radiation but it is also true for all the moving material particles of the universe.

- It means that like radiation, matter also have wave–particle duality. The wavelength of the matter-wave is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

EXPRESSION FOR WAVELENGTH OF MATTER-WAVE (de BROGLIE WAVELENGTH)

- A light wave of frequency ν is associated with a photon of energy E. By applying Planck's relation, E = hν.
- According to Einstein's theory of relativity, a particle of mass m is equivalent to energy mc^2, i.e., E = mc^2
- If a photon attains a mass m during its motion with velocity c, then, hν = mc^2

EXPRESSION FOR WAVELENGTH OF MATTER-WAVE

- Since the momentum of photon is p = mc, using this relation in above equation, $h\nu = pc$

- $= p\nu\lambda$ (because $c = \nu\lambda$)

$$\lambda = \frac{h}{p}$$

Example: de Broglie wavelength of an electron

- Mass = 9.11 x 10^{-31} kg
  Speed = 10^6 m / sec

$$\lambda = \frac{6.63 \times 10^{-34} \text{Joules} \cdot \text{sec}}{(9.11 \times 10^{-31} \text{ kg})(10^6 \text{ m/sec})} = 7.28 \times 10^{-10} \text{m}$$

This wavelength is in the region of X-rays

Example: de Broglie wavelength of a ball

- Mass = 1 kg
  Speed = 1 m / sec

$$\lambda = \frac{6.63 \times 10^{-34}\,\text{Joules} \cdot \text{sec}}{(1\,\text{kg})(1\,\text{m/sec})} = 6.63 \times 10^{-34}\,\text{m}$$

PROPERTIES OF MATTER-WAVES

- The lighter particles have greater wavelength than the heavier particles.

- The smaller the velocity of the particle, the greater is the wavelength $\lambda = h/mv$ associated with it.

- From the expression of the de Broglie wavelength, i.e., λ = *(h/mv, if $v = 0$, then $\lambda = \infty$, whereas if $v = \infty$, then $\lambda = 0$. This shows that the matter-waves are generated only when the particle is in motion.*

PROPERTIES OF MATTER-WAVES

- The matter-waves are independent of the charge. Thus, they are produced by both charged and uncharged particles. This shows that the matter-waves are not electromagnetic waves; they are entirely different waves.

- The velocity of the matter-waves is not constant. It depends on the velocity of the particle, while the velocity of the electromagnetic waves is constant.

- The velocity of the matter-waves may be greater than the velocity of light.

PROPERTIES OF MATTER-WAVES Contd..

- Wave velocity of a matter-wave is given in terms of group velocity as

$$\upsilon_{\text{phase}} = \frac{c^2}{\upsilon_{\text{group}}}$$

$$E = h\nu$$

Also,

$$E = mc^2$$

or $$h\nu = mc^2$$

or $$\nu = \frac{mc^2}{h}$$

The wave velocity (ω) is given by

$$\omega = \nu \times \lambda$$

$$= \frac{mc^2}{h} \frac{h}{m\upsilon} = \frac{c^2}{\upsilon}$$

Since a particle cannot travel with the velocity more than the velocity of light, thus, w will be greater than the velocity of light. The satisfactory explanation of such unexpected result can be explained with the help of wave velocity and group velocity.

PROPERTIES OF MATTER-WAVES

- where u_{group} is equal to the particle velocity (see Section 22.15). Since a particle cannot travel with the velocity more than the velocity of light, the velocity of matter-wave will be greater than c.

- Wave–particle duality (wave nature of matter) introduces the concept of uncertainty. This concept suggests that if the particle nature of matter becomes certain, the wave nature will be uncertain and vice versa.

Assignment based what we learnt in this lecture

- Compare the properties of wave and particles
- What is de- Broglie hypothesis of wave particle duality?
- Obtain the expression for de-Broglie wavelength
- Write the expression of de-Broglie wavelength in different form
- Mention the properties of Matter wave
- Obtain the relation between wave velocity and group velocity
- Explain the wavelength of waves associated with matter using suitable numerical problems.