MPM: 203 NUCLEAR AND PARTICLE PHYSICS

UNIT –I: Nuclei And Its Properties

Lecture-9

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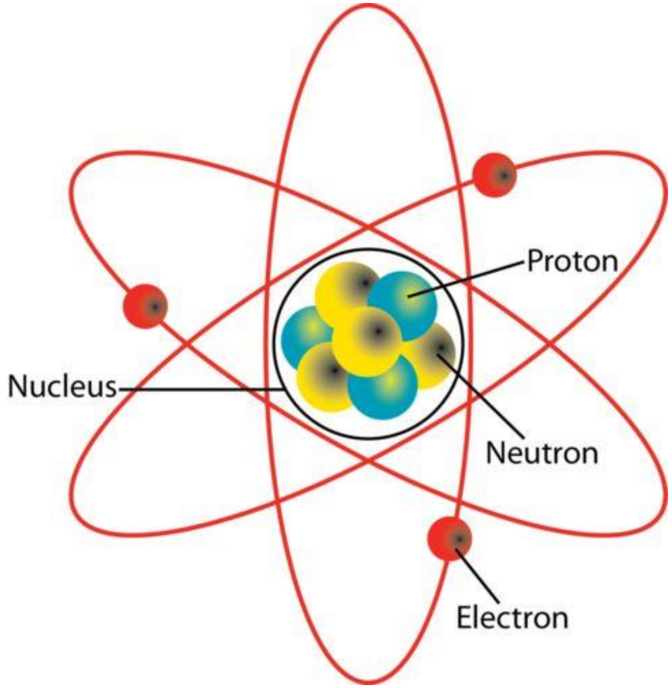
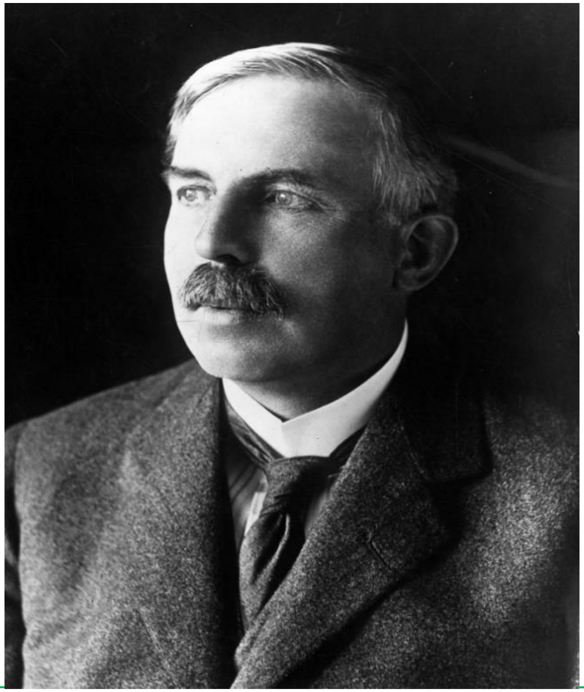




Nuclear force Range and Strength

- **Structure of Nucleons(Z and N)**
- **Characteristics of Nucleons**
- **Forces between nucleons**
- **Deuteron Case**
- **Schrodinger wave equation for deuteron**
- **Solution of Wave equation**
- **Conclusion**

Structure of Nucleons(Z and N)

- Scattering cross section of nucleus suggest that there is empty region in side the nucleus.
- Positive charge is concentrated in the nucleus containing neutron and proton.
- Now the question is what is inside the nucleons that is inside the proton and neutrons
- In order to know it high energy electron (GeV) bombarded on nucleons.
- The scattered electrons and its angular distribution reveals the fact that there is certain structure inside the nucleus.

Structure of Nucleons(Z and N)

- There are point particles inside the nucleons known as quark particles.

- Protons are made up of three quarks
- Neutrons are also made of three quarks

- Actually the entire mass of the universe is mad of two kinds of particles known as quarks and Leptons.

- The Quarks are of six types with their six antiparticles
- **Up Down Strong Charm Top Bottom**
- Charges of these quark particles are
- $\frac{2}{3}e$ $-\frac{1}{3}e$ $-\frac{1}{3}e$ $\frac{2}{3}e$ $\frac{2}{3}e$ $-\frac{1}{3}e$

Structure of Nucleons(Z and N)

- Constitution of Proton and Neutron
- **Proton---- U U D** = $\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = e$
- **Neutron---- D D U** = $-\frac{1}{3}e - \frac{1}{3}e + \frac{2}{3}e = 0$
- **These quarks have different colours but the net colour in the constitution of proton and neutrons are nil.**

Characteristics of Nucleons

- Net charge of the neutron and proton is the sum of its constituent quark particles
- The force between nucleons is known as nuclear force
- Actually the nuclear forces are the forces between the charge particles constituting the nucleons.
- These forces are strong forces
- **Gravitational force is governed by --------mass of particles**
- **Electromagnetic force is governed by --------charges**
- **Strong nuclear forces are governed by -------colours**

Characteristics of Nucleons

- Like the intrinsic properties of particles as mass and charge, colour is the intrinsic properties of quarks
- Actually there are three quarks constituting the proton and neutron have different colours but when they recombine become colourless,
- Initial colours of these quarks are red green blue (RGB)

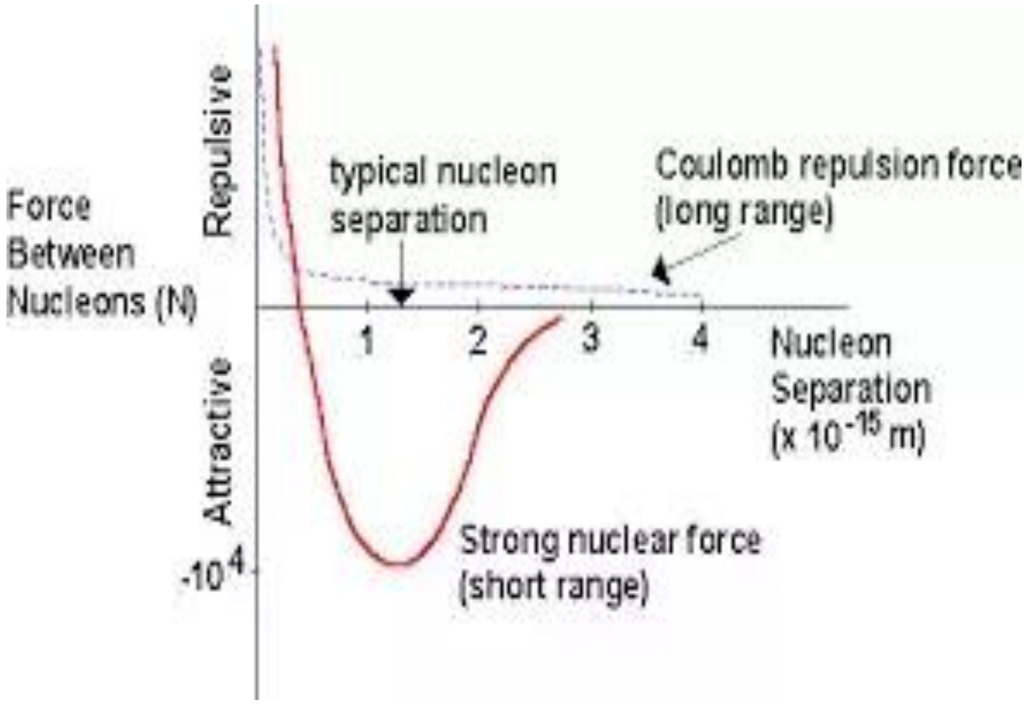
Forces between nucleons

- Now there is interesting question **" If net colour of combined quark particles is nil then how strong nuclear force appears?"**

- Ans: **When these nucleons are for apart then there is no interaction between its constituent particles but when they become very close there is interaction between their constituent particles. May be UU interact and DD interact to contribute some net value which results the strong nuclear force.**

Forces between nucleons

- Molecular potential energy is shown in the Fig.
- This short range interaction force is due to the colour- colour interaction of nucleon constituents.



- Now we have to find the range and strength of these short range forces.

Deuteron Case

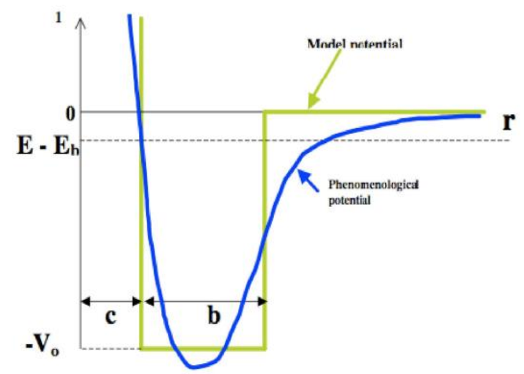
- Let the case of Deuteron having one proton and one neutron which is the simplest case of nucleons.
- ${}^{2}_{1}H_{1}$ Where A=2, Z = 1, N=1.
- With the above Combination of n-p deuteron is a bound system.
- Binding energy is – 2.225 MeV ; it is a weakly bound system because BE/ Nucleon is 1.112 MeV while most of the atoms have BE/N is about 6-7 MeV and range is 2.1 Fm
- Now we have to calculate the strength of Nuclear force????

Deuteron Case

- Nature of nuclear force in case of deuteron is very complicated, to simplify this case using some assumption of square well potential with E= -2.225 MeV and the range is 2.1 Fm.

- Now with this assumption we will find the strength or depth of potential.

Deuteron Wave function



Nuclear model potential for studying the deuteron

To simplify the analysis of the deuteron, we assume that the nucleon-nucleon potential is *a three-dimensional square well*, as shown in the figure ***a***:

- E is different from -V_0 , because -V_0 is different from the binding energy i.e. nuclear interaction energy.

Schrodinger wave equation for deuteron

- Deuteron have no excited state energy.
- It have single bound state at 2.225 MeV as zero energy state.
- This three dimensional potential changes in all the directions as r changes.
- This is a potential is a kind of spherical volume region.
- The assumed potential is a central potential.
- V($\overrightarrow{r}$) = V(r) does not depend on θ and $\emptyset$ where r is the separation between particles.

Schrodinger wave equation for deuteron

- Net mass of the system is given as
- m= $\frac{M_1 M_2}{M_1+M_2}$; Reduced mass of the particle system

- For Eigen value we use time dependent Schrodinger wave equation as
- Hψ (r) = Eψ (r)

- $\left[\frac{-\hbar^2}{2m}\nabla^2 + V(r)\right]$ ψ (r) = Eψ (r)

- Since the system is the spherical symmetric system so E will not depend on θ *and* $\emptyset$.

Schrodinger wave equation for deuteron

- ψ (r) = ∅ (r) $Y_l^{m_l}(\theta, \emptyset)$ In this expression only radial part will decide the energy second part is fixed. This part represent the spherical harmonics l= 0,1,2,….. m_l = 0, $\pm$1, $\pm$2, ….. $\pm$l.

- Let ∅(r) = $\frac{u(r)}{r}$

- Now, $\frac{-\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V(r) + \frac{l(l+1)\hbar^2}{2mr^2}\right]$u = Eu

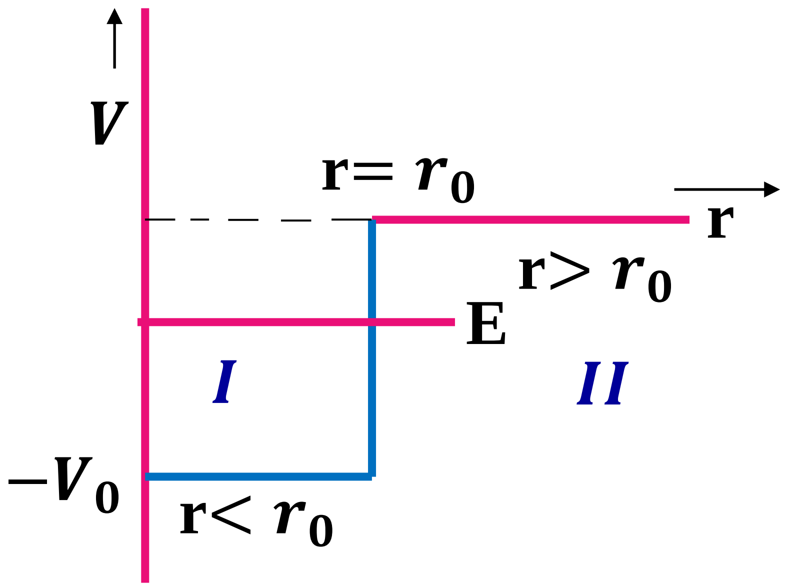
- The deuteron have only one energy level so the allowed value of l is l=0.

Schrodinger wave equation for deuteron

- Now the above equation will be given as
- $\frac{-\hbar^2}{2m}\frac{d^2u}{dr^2} + [V(r)]u = Eu$
- Boundary condition for $\psi(r)$ thus for u(r) can be given as
- It should be continuous
- Finite everywhere
- Square integrable --$\int_0^\infty \psi^2\, d\tau$ = finite, thus $\int_0^\infty u^2\, d\tau$ = finite
- $\frac{d\psi}{dr}$ = should be continuous

Solution of Wave equation

- There are two regions shown in the figure for I region r< r_0; in this region v= $-V_0$
- Thus the equation can be given as



- $\frac{-\hbar^2}{2m}\frac{d^2u}{dr^2}$ - $V_0\, u = Eu$
- $\frac{-\hbar^2}{2m}\frac{d^2u}{dr^2} = (V_0 + E)u$
- $\frac{d^2u}{dr^2} = -\left[\frac{2m}{\hbar^2}(V_0 + E)\right]u$

Solution of Wave equation

- $\frac{d^2u}{dr^2}$ = - k^2u Where k = $\left[\frac{2m}{\hbar^2}\left(\left[\frac{2m}{\hbar^2}(V_0 + E)\right] + E \right)\right]$

- Here V_0 is positive E is negative but E is less in magnitude than V_0.

- For r< $\boldsymbol{r_0}$
- u (r) = A Sin kx + B Cos k r

- $\frac{\text{u (r)}}{r}$ = ∅ (r) → u (r) = r ∅ (r) where ∅ (r) is finite everywhere
- At r = 0, u_I(r) = 0 → B = 0, If r = 0 → r ∅ (r) = 0

- u_I(r) = A Sin kr ------(1)

Solution of Wave equation

- **<u>Case II where r $> r_0$ and V = 0</u>**
- Now the wave equation will
- $\frac{-\hbar^2}{2m}\frac{d^2u}{dr^2} = Eu$
- $\frac{d^2u}{dr^2} = -\left[\frac{2m \quad E}{\hbar^2}\right]u = \gamma^2 u\ ;\ \gamma = \sqrt{\frac{-2mE}{\hbar^2}} \rightarrow \gamma$ *is real*
- E *is negative because it is a bound system*

Solution of Wave equation

- *Now*
- u_{II}(r) = C $e^{\gamma r}$ + D $e^{-\gamma r}$
- → *beyond* r $> r_0$ it is an exponential decay and rise
- *Boundary condition square integrable*
- $\int_0^\infty \psi^2 \, d\tau$ = *finite* → $\int_0^\infty u^2 \, d\tau$ = *finite if r → ∞ then u → 0 otherwise integral will not be zero.*
- *At r → ∞ then u → 0 thus c=0 hence,*
- u_{II}(r) = D $e^{-\gamma r}$-------*(2)*

Conclusion

- Applying the other boundary conditions
- At the boundary r $= r_0$, the value of u in first and second region should be same because it should be continuous at the boundary.
- Thus,
- $u_I(r_0) = u_{II}(r_0) \rightarrow$ A Sin kr_0 = D $e^{-\gamma r_0}$ -------(3)
- Similarly
- Ak cos kr_0 = $-\gamma$ D $e^{-\gamma r_0}$ ---------------(4) $[\frac{du_I(r_0)}{dr} = \frac{du_{II}(r_0)}{dr}]$
- From equation (3) and (4) we get

Conclusion

- k cot kr_0 = $-\gamma$
- Or $\sqrt{\frac{2m}{\hbar^2}(V_0 + E)}$ cot $r_0\sqrt{\frac{2m}{\hbar^2}(V_0 + E)}$ = $-\sqrt{\frac{2mE}{\hbar^2}}$
- r_0 = 2.1 Fm and E = -2-225 MeV
- M= $\frac{m_p m_n}{m_p + m_n}$
- By putting the values of known quantities we can get the value of V_0
- V_0 = 36 MeV
- Now the strength of potential can be imagine that how strong is the nuclear force.