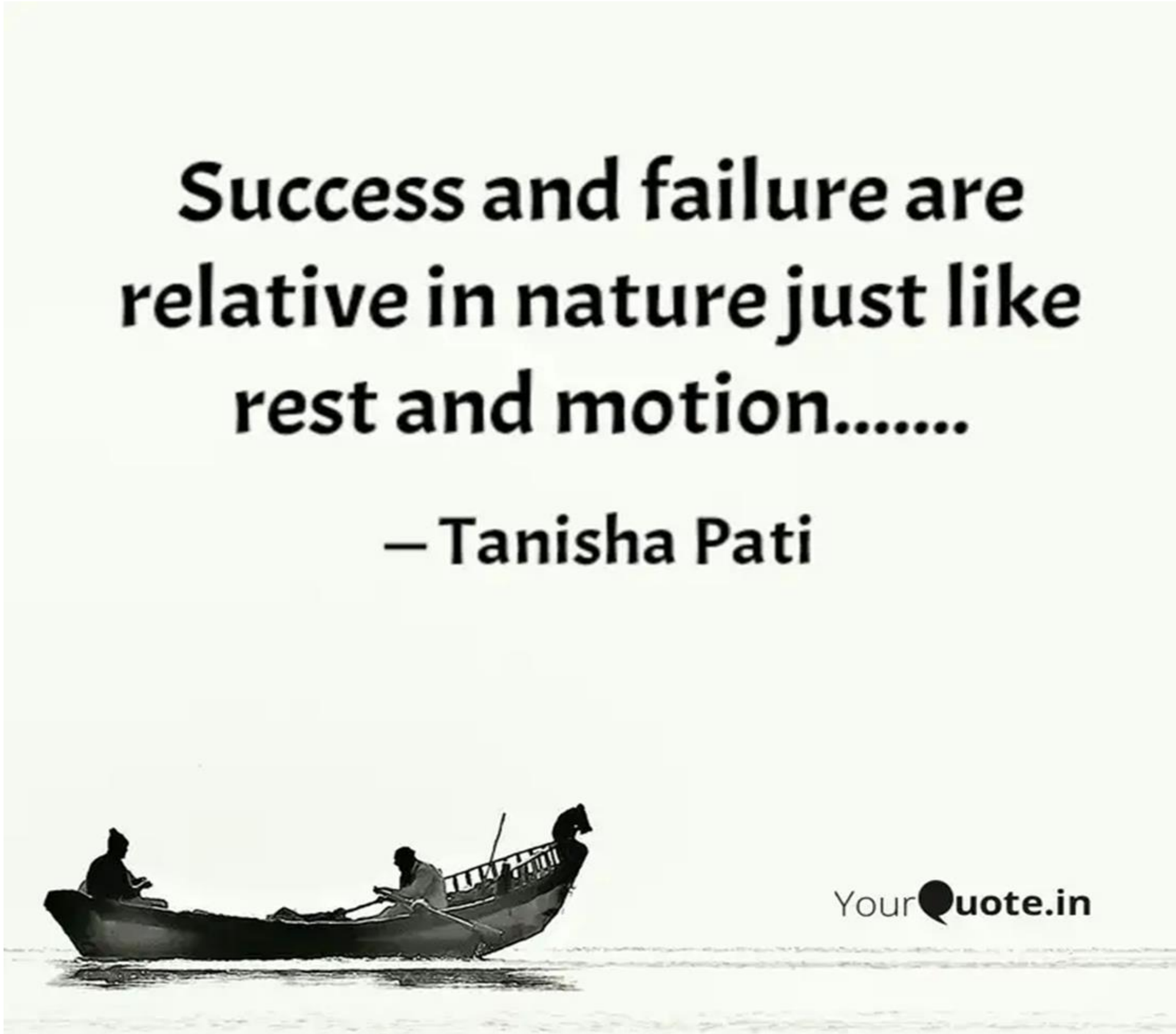
Theory of Relativity

UNIT I
Relativistic Mechanics

Lecture-1

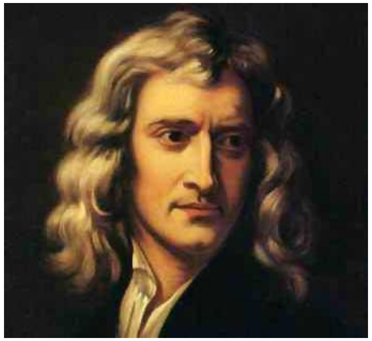

Success and failure are
relative in nature just like
rest and motion.......
– Tanisha Pati
YourQuote.in

Theory of Relativity

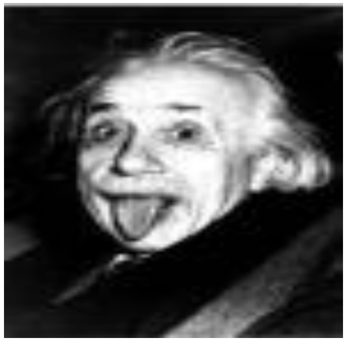
- All the measurements in this universe are relative
- According to special theory of relativity , the fundamental quantities, i.e., space, time, and mass no longer remain constant or invariant but depend on the state of motion.
- The theory of relativity predicts that the *Newtonian mechanics is the limiting* case of the *special theory of relativity.*
- The theory of relativity deals with the way in which observers in relative motion describe the physical phenomenon with respect to one another.

Time is the 4th dimension

Einstein discovered that there is no "absolute" time, it too depends upon the state of motion of the observer



Space and Time



completely different concepts

2 different aspects of the same thing

Concept of Theory of Relativity

- Complete information of an event can be obtained by knowing the facts about where the event is occurring (position), what is its time of occurrence (when), and who is recording the data for it.

- According to the choice of the position or reference frame same observer may detect same event in different ways as by being at center of a wheel and on the ground the mark on a moving rim will appear as circle and cycloid respectively.

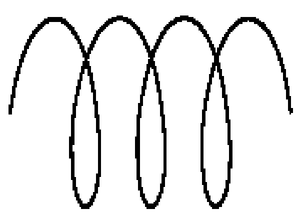


Fig. 1.1(a) Path of point on the rim when observer is on the ground

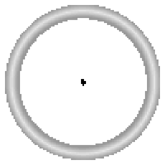


Fig. 1.1(b) Path of point on the rim when observer is at the centre of the wheel

Frames of reference

- If we imagine a coordinate system attached to a rigid body and describe the position of any particle in space relative to it, then such a coordinate system is known as *frame of reference*.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

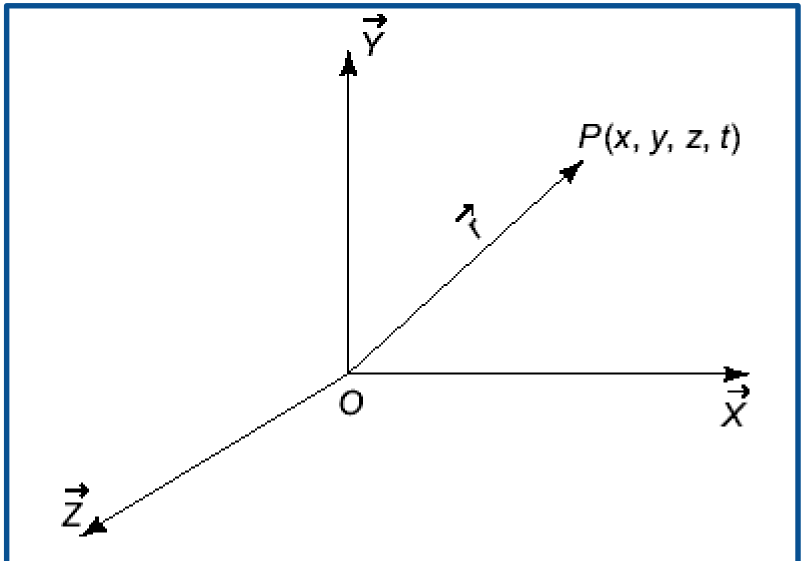
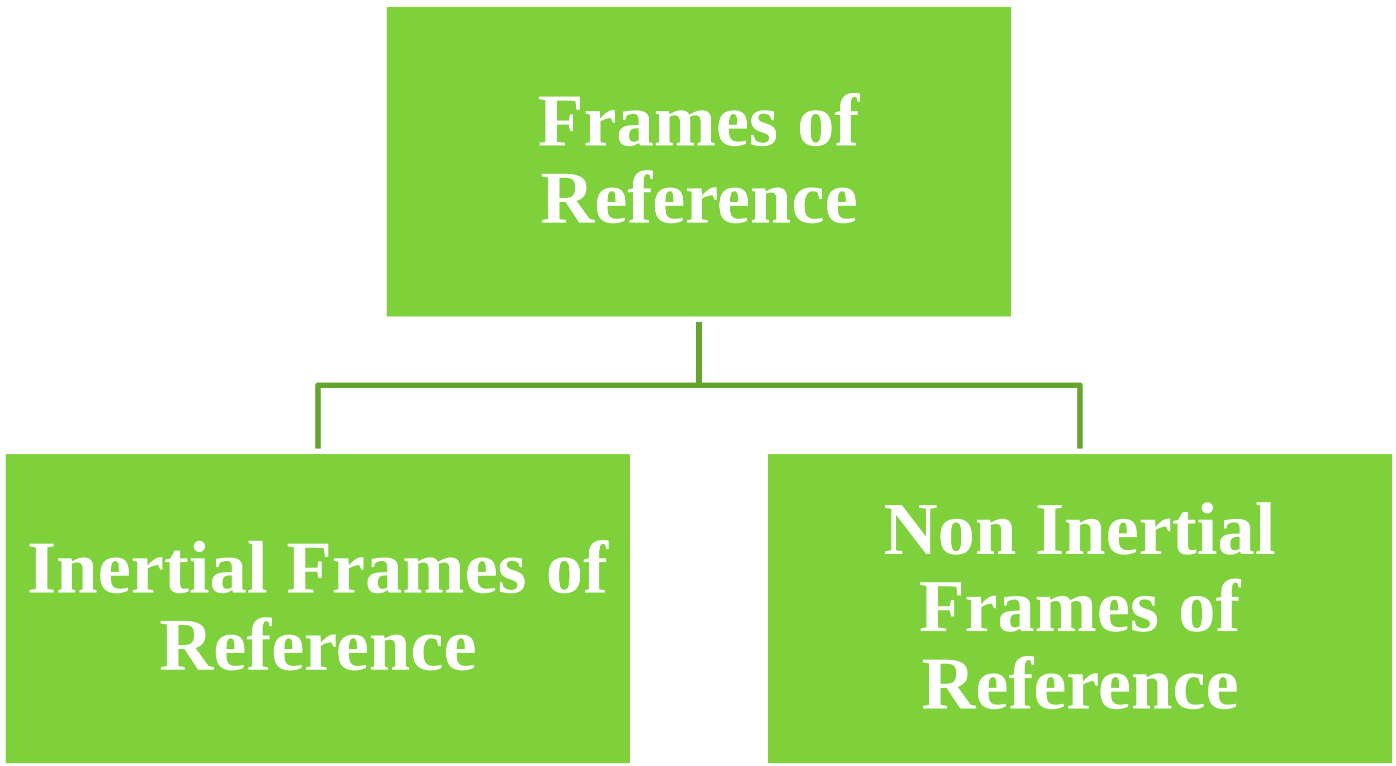


Fig. 1.2 Space–time frame of reference

Frames of References

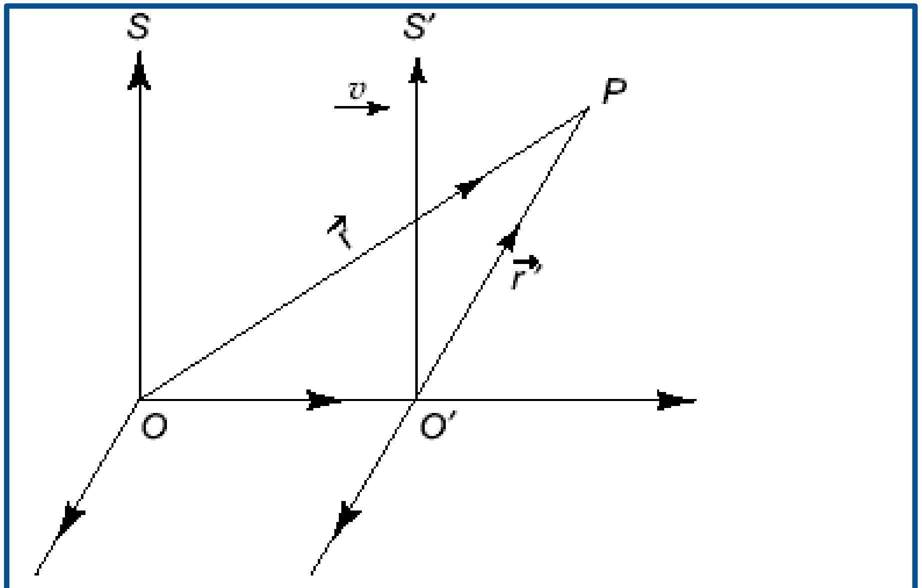


Inertial Frames of reference

- The inertial frames of reference are those unaccelerated frames of reference in which Newton's laws hold good.

- In an inertial frame of reference, a body does not experience any external force; therefore, its acceleration *a is given by*

$$\vec{a} = \frac{d^2 \vec{r}}{dt^2} = 0 \quad \text{(Because } \vec{F} = m\vec{a} = 0\text{)}$$



S and S′ are intertial frames of reference

Non Inertial Frames of Reference

- Those accelerated frames of reference, in which Newton's laws do not hold good are called *non-inertial frames of reference.*

- Suppose that S *is an inertial frame and another frame S' is moving with an acceleration* a_0 *relative to S.*

- *The acceleration of a particle P, on which no external force is acting, will be zero in the frame S, but in* frame *S ', the observer will find that an acceleration* $-a_0$ *is acting on it.*

- *Thus, in frame S', the observed* force on the particle is $-ma_0$, *where m is the mass of the particle. Such a force, which does not really* act on the particle but appears due to the acceleration of the frame, is called a *fictitious, or pseudo force.*

- Here, fictitious force on the particle P *is* $F = -ma_0$.

Conceptual Questions

Example 1.1

Calculate the fictitious force and the total force on a body of mass 6 kg if the frame of reference is moving (i) vertically upwards and (ii) vertically downwards, with an acceleration of 5 m/s^2.

Solution

$$\text{Weight of the body} = mg = 6\,(-9.8) = -58.8 \text{ N} = 58.8 \text{ N downwards}$$

(i) The fictitious force acting on it, when the frame of reference is moving in upward direction is given as

$$-ma_0 = -6 \times 5 = -30 \text{ N} = 30 \text{ N downwards}$$

Hence, the total force = 58.8 + 30 = 88.8 N downwards, i.e., the body appears to be heavier.

(ii) The fictitious force acting on the body during the downward motion

$$= -ma_0 = 6 \times (-(-5)) = 30 \text{ N upwards}$$

The net force experienced by the body = 58.8 – 30 = 28.8 N downwards so that it seems to be lighter.

Conceptual Questions

The earth, revolving around its own axis, is not an inertial frame of reference. If, however, we take it to be so, what would be the error involved in 1 s in the position of a particle close to its surface? (Given that radius of the earth = 6.4×10^8 cm.)

Solution

The particle which is close to the surface of the earth will experience a centripetal acceleration due to the rotation of the earth about its own axis, whose value can be given as

$$\alpha = \omega^2 R$$

$$= \left(\frac{2\pi}{24 \times 60 \times 60}\right)^2 \times 6.4 \times 10^8 \text{ cm/s}^2$$

$$= 3.4 \text{ cm/s}^2 \quad \left(\text{because } \omega = \frac{2\pi}{T}\right)$$

Thus, the acceleration of the particle will differ by 3.4 cm/s^2 when it will be measured in true inertial frame of reference. Due to this acceleration on the surface of the earth, the particle will be altered by a distance $1/2\ at^2$ in time t.

Now, the altered distance S for the particle in 1 s can be given as

$$S = \frac{1}{2}\ at^2$$

$$= \frac{1}{2} \times 3.4 \times (1)^2$$

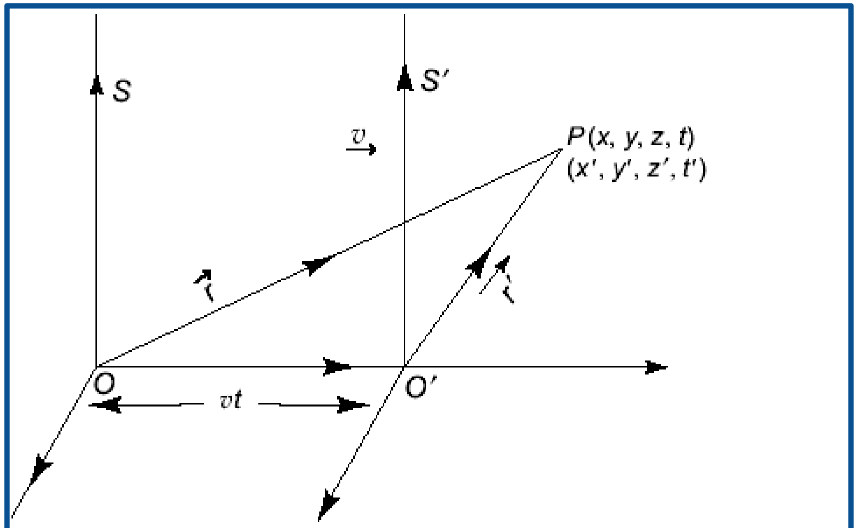
$$= 1.7 \text{ cm}$$

Hence, the error involved in the position of the particle measured in earth's frame will be 1.7 cm in 1 s.

Galilean Transformation Equations

- When a point or an event is observed from two different frames, then it has different coordinates. There are certain equations that can relate these coordinates. These equations are known as *transformation equations.*
- ***Case I :*** When second frame of reference is moving only along X-axis then the transformation equations can be given as

$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\}$$



Relative motion of S′ with respect to S along x-axis

Galilean Transformation Equations

Case II **:** In this case, we will derive the transformation equations between the coordinates of two inertial frames of reference, where the motion of the second frame of reference is along all the directions (*x*, *y*, *z*) *simultaneously.*

Galilean transformation equations for such frames of reference which have their relative motion along all the three axes simultaneously can be given as .

$$\left.\begin{aligned} x' &= x - v_x t \\ y' &= y - v_y t \\ z' &= z - v_z t \\ t' &= t \end{aligned}\right\}$$

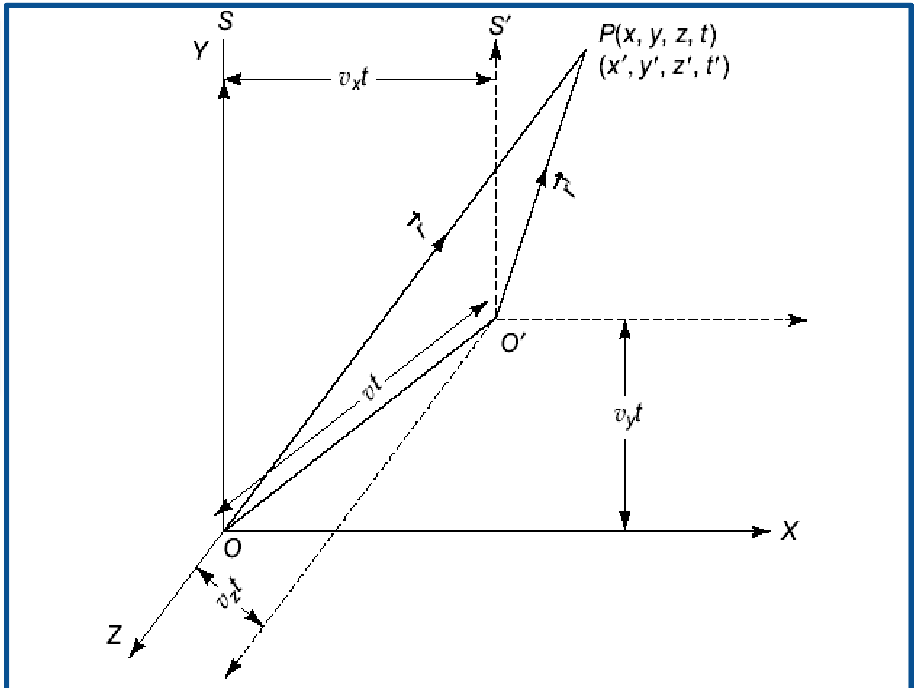


Fig. 1.5 Relative motion of S′ with respect to S along all the axes

Concept of "Luminiferous Ether" Medium

- For the propagation light and other electromagnetic waves in the free space an hypothetical medium is proposed.

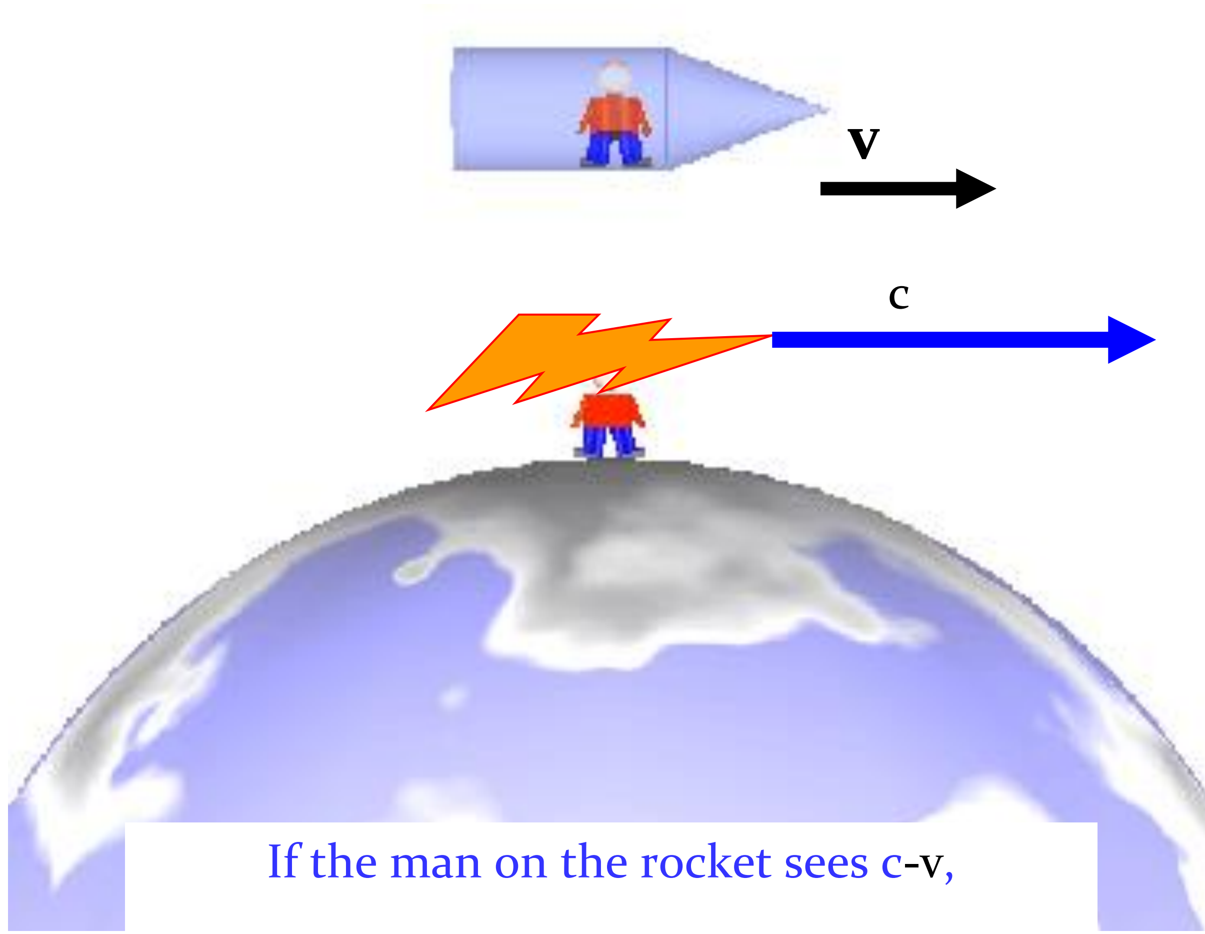
- This hypothetical universal ether medium is rigid, invisible, massless, perfectly transparent, perfectly non-resistive, continuous, and stationary solid like steel having a very high elasticity and negligible density.

- All the objects including the earth, moon, stars, etc., may move freely through this hypothetical ether medium without any disturbance.

- Thus, it may be concluded that ether provides a universal fixed frame of reference or absolute frame of reference with respect to which all the measurements can be done.

Search of Absolute Frame of Reference ???

Watching a Light Flash go by



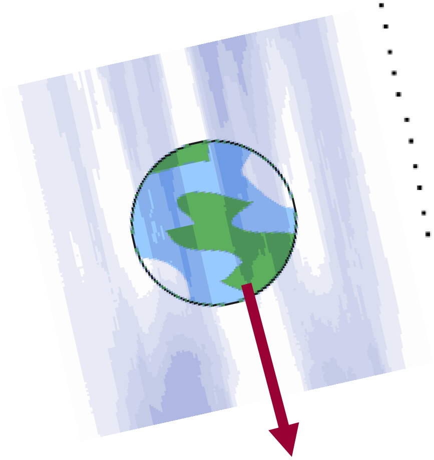
If the man on the rocket sees c-v,

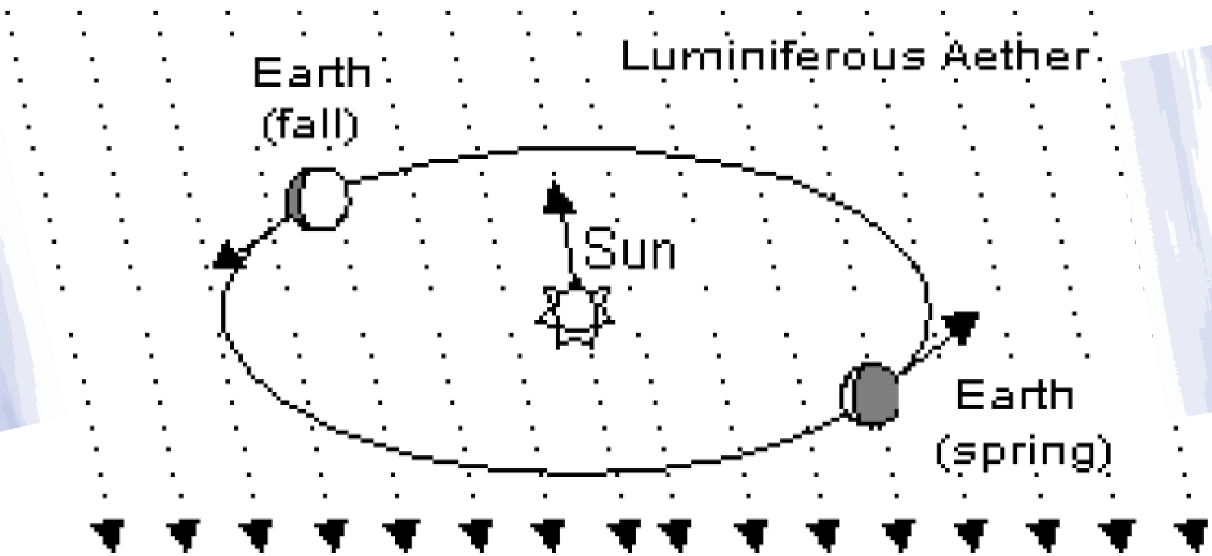
He disagrees with Maxwell

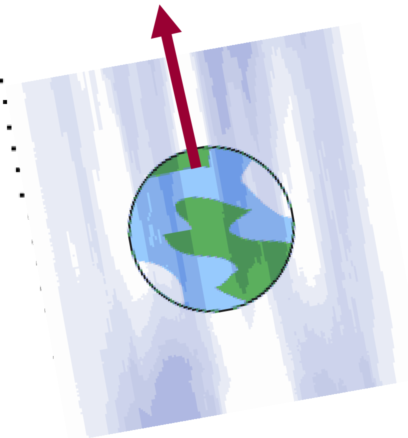
Do Maxwell's Eqns only work in one reference frame?

If so, this would be the rest frame of the luminiferous Ether Medium

If so, the speed of light should change throughout the year







Relative velocity between earth and light is c-v

Relative velocity between earth and light is c+v

What you have learnt ?

- Define relativity?
- Define an event?
- How same event appears different when observed from different positions?
- What do you understand by frames of references?
- Differentiate between inertial and non inertial frames of reference.
- What do you mean by transformation equations?

Assignment based on what we learnt in this lecture?

- Find the Galilean transformation of position, velocity and acceleration.
- Show that acceleration remains constant under Galilean transformation,
- Show that the length of a rod remains invariant under Galilean transformation.