MPM: 203 NUCLEAR AND PARTICLE PHYSICS

UNIT –I: Nuclei And Its Properties

Lecture-5

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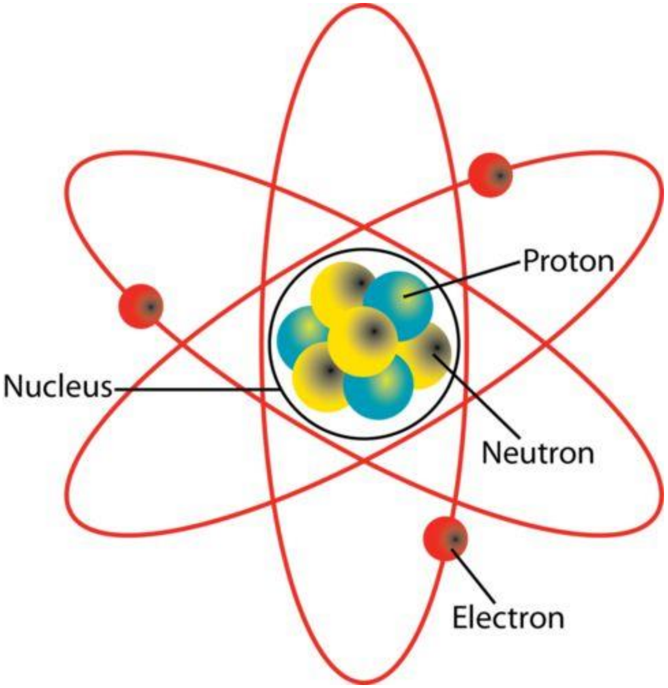
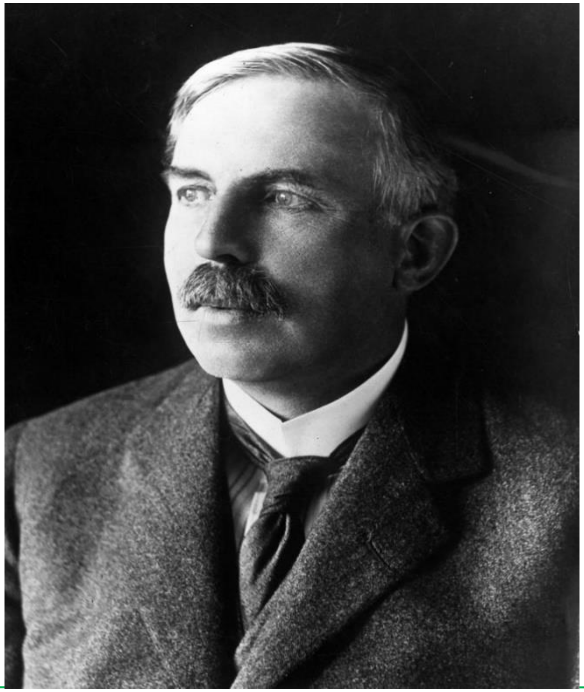




Nuclear Magnetic Moment

- The Magnetic dipole moment produced by a current loop is given as

  $\boldsymbol{\mu} = iA$

Where I is the current and A is the area of the loop

- A spinless particle having charge e revolving in a loop will set a magnetic dipole moment. If particle revolves with velocity v in a circular orbit of radius r, then equivalent current
- i= $\frac{Charge}{time\ period} = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$
- Magnetic dipole moment = $\boldsymbol{\mu}_l$ = iA= $\frac{ev}{2\pi r}\pi r^2$, i.e. $\boldsymbol{\mu}_l = \frac{evr}{2}$
- The orbital angular momentum of the particle L= mvr
- The ratio of magnetic dipole moment to the orbital angular momentum is called the gyro-magnetic ratio i.e.
- $\gamma = \frac{\mu_l}{L} = \frac{evr/2}{mvr} = \frac{e}{2m}$

Nuclear Magnetic Moment Contd..

- The relation will also hold for the components of two vectors $\overrightarrow{\mu_l}$ and $\vec{l}$ along any direction (in the direction of magnetic field)
- Z- component of dipole magnetic moment
- $(\mu_L)_Z = \frac{e}{2m} L_Z = \frac{e}{2m} m_l\hbar$
- $(\mu_L)_Z = \frac{e\hbar}{2m} m_l$
- This is the relation for the magnetic dipole moment along the direction of magnetic field due to the orbital motion of electron in an atom.
- By analogy same orbital motion may hold for proton inside the nucleus.
- For the proton, the mass m is replaced by proton mass m_p i. e. dipole moment of proton in the nucleus along Z- direction is
- $(\mu_L)_Z = \frac{e\hbar}{2m_p} m_l$

Nuclear Magnetic Moment Contd..

- The quantity $\frac{e\hbar}{2m_p}$ forms the natural unit of nuclear magnetic magnetic moment; it is called nuclear magneton.
- i. e. 1 nuclear magneton (nm), $\mu_n = \frac{e\hbar}{2m_p}$ = 5.0505 x 10^{-27} Joule/tesla.
- Clearly the Z- component of the magnetic dipole moment resulting from orbital motion of a proton is just m_l times nuclear magnetons.
- Maximum value of m_l is l; therefore magnitude of maximum value of $(\mu_L)_Z$ is given as
- $(\mu_L)_{Zmaximun} = \frac{e\hbar}{2m_p} l$

Nuclear Magnetic Moment Contd..

- Like electron, proton also spins about its axis; therefore it also possesses magnetic dipole moment due to spin motion
- The magnetic dipole moment due to spin motion of proton is
- $(\mu_S)_Z = \frac{e\hbar}{2m_p} m_s$
- Where m_s can take only two values $\pm\frac{1}{2}$
- Neutron also possesses magnetic dipole moment due to the spin motion.
- Total angular momentum of nucleus is the contribution due to orbital and spin motions, therefore the total magnetic dipole moment of a nucleus is the vector sum of magnetic dipole moments due to orbital and spin motions of protons and due to spin motion of neutron.

Nuclear Magnetic Moment Contd..

- The Magnetic dipole moment is conveniently written as

$$\vec{\boldsymbol{\mu}} = \boldsymbol{g_l \mu_N \vec{I}}$$

- Where $\boldsymbol{g_l}$ is nuclear g- factor and $\vec{\boldsymbol{I}}$ is the total angular momentum .

- In the vector form $\vec{\boldsymbol{I}} = \vec{\boldsymbol{L}} + \vec{\boldsymbol{S}}$.
- The maximum observable component of $\vec{\boldsymbol{\mu}}$ is known as the magnetic moment of the nucleus and is expressed as $\mu = \boldsymbol{g_l \mu_N}$I.

- It may be noted that it is the value of nucleon magnetic moment which is measured experimentally is about $\frac{1}{1000}$ of the electromagnetic moment.

- It implies that the electrons can not be the part of nuclear constituents.

Electric Quadrupole Moment

- The electric dipole moment measures the departure of the a nucleus from spherical symmetry.
- Usually the nucleus is assumed spherically symmetrical.
- But it is not always necessary to make this assumption.
- Any nucleus in a stationary state does not possess a dipole moment because the centre of charge and centre of mass can be assumed to coincide with one another.
- The electric quadrupole moment of a nucleus is calculated as follows from the classical consideration.
- Let us consider that the charge is not situated at the centre of the nucleus (departure from spherical symmetry) but is located at P' having rectangular co-ordinates (x, y, z).

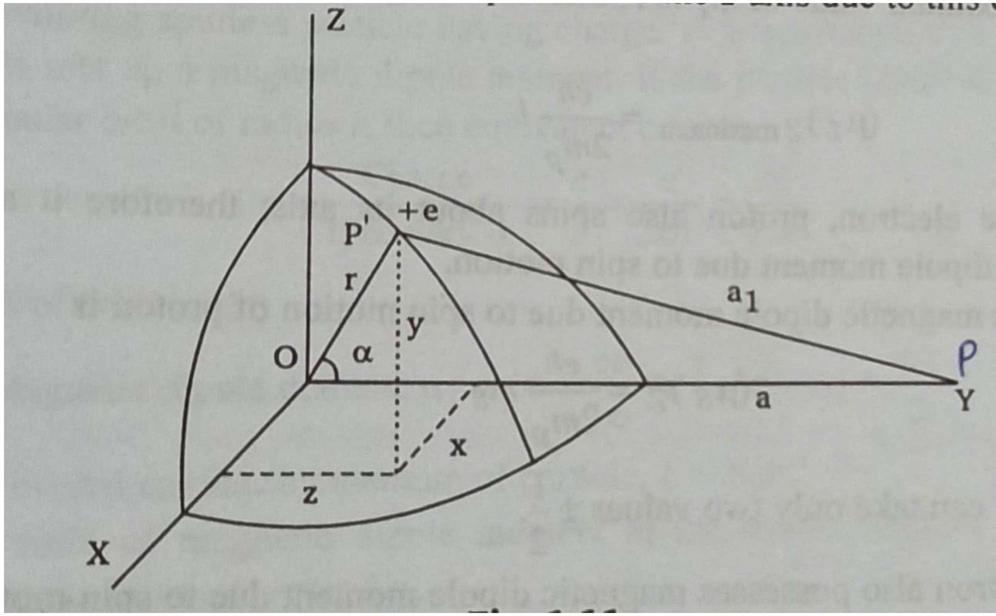
Electric Quadrupole Moment

- The potential at a point P on the z-axis due to this charge is

- $\boldsymbol{\phi} = \frac{1}{4\pi\epsilon_0}\frac{e}{a_1}$ ….. **(1)**

Where

$a_1 = (a^2 + r^2 - 2arcos\alpha\ )^{1/2}$ ..(2)



- r is the distance of the charge from the origin and is given by
- $a_1 = (x^2 + y^2 + z^2)^{1/2}$ and $cos\alpha = \frac{z}{r}$, defines the angle between a and r.
- Now the equation of the potential can be given as
- $\boldsymbol{\phi} = \frac{1}{4\pi\epsilon_0}\frac{e}{(a^2+r^2-2arcos\alpha\ )^{1/2}}$

Electric Quadrupole Moment Contd…

- $\boldsymbol{\phi} = \frac{1}{4\pi\epsilon_0}\frac{e}{a}\left(1 - 2\frac{r}{a}cos\alpha - \frac{r^2}{a^2}\right)^{-1/2}$ ------(3)
- *We know*
- $(1 - x\,)^{-1/2} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \ldots\ldots$
- Thus
- $\boldsymbol{\phi} = \frac{1}{4\pi\epsilon_0}\left[\frac{e}{a} + \frac{e}{a^2}rcos\alpha + \frac{e}{a^3}r^2\left(\frac{3}{2}cos^2\alpha - \frac{1}{2}\right) + \frac{e}{a^4}r^3\left(\frac{5}{2}cos^3\alpha - \frac{3}{2}cos\alpha\right)\right]$-(4)
- $\boldsymbol{\phi} = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{er^n}{a^{n+1}}P_n(\cos\alpha)$----------(5)
- Where $P_n(\cos\alpha)$ are the Legendre polynomials and n is the multiple order.
- In equation (4) the coefficient of $\left(\frac{1}{a}\right)$ is known as monopole strength, the coefficient of $\left(\frac{1}{a^2}\right)$ is the z-component of the dipole moment, the coefficient of $\left(\frac{1}{a^3}\right)$ is the Z- component of quadrupole moment, the coefficient of $\left(\frac{1}{a^4}\right)$

Electric Quadrupole Moment Contd…

- In equation (4) the coefficient of $\left(\frac{1}{a}\right)$ is known as monopole strength, the coefficient of $\left(\frac{1}{a^2}\right)$ is the z-component of the dipole moment, the coefficient of $\left(\frac{1}{a^3}\right)$ is the Z- component of quadrupole moment, the coefficient of $\left(\frac{1}{a^4}\right)$ is the z-component of octupole moment etc.
- The first term in the above expression is the ordinary Coulomb potential.
- Thus the nucleus possesses a net electric quadrupole moment given by
- $Q = er^2\left(\frac{3}{2}cos^2\alpha - \frac{1}{2}\right) = \frac{er^2}{2}(3\ cos^2\alpha\text{-}1)$
- Putting $cos\alpha = \frac{z}{r}$ we get
- $Q = \frac{e}{2}(3\ z^2\text{-}\ r^2)$

Discussion

- According to the expression Q = $\frac{e}{2}$ (3 z^2- r^2)
- The quadrupole moment is zero if the charges are evenly placed about all three axes.
- Thus the quadrupole moment for a spherically symmetric charge distribution is zero.
- Deviation from spherical charge distribution nuclei creates quadrupole effect.
- If Q is positive charge, nuclei are elongated along Z-axis while if Q is negative, the nuclei are contracted along z axis.
- The unit of quadrupole moment is coulomb x barn (1 barn = $10^{-28}\,m^2$)
- Quadrupole moment can be estimated by variety of processes which involve interaction between the nucleus and the applied field or the field due to atomic electron.

Discussion

- All the nuclei are not spherical in shape, some are prolate ellipsoids while some are oblate ellipsoids.
- The nuclei having N (number of neutrons) or Z (Number of Protons) values 2, 8, 20, 28, 50, 82, 126 are spherical in shape.
- Quadrupole moments mostly lie in the range 10^{-26} to 10^{-23} cm^2, which is of the order of nuclear area.
- The nuclear size is of the order of 1.2 $A^{1/3}$ x 10^{-15}m.