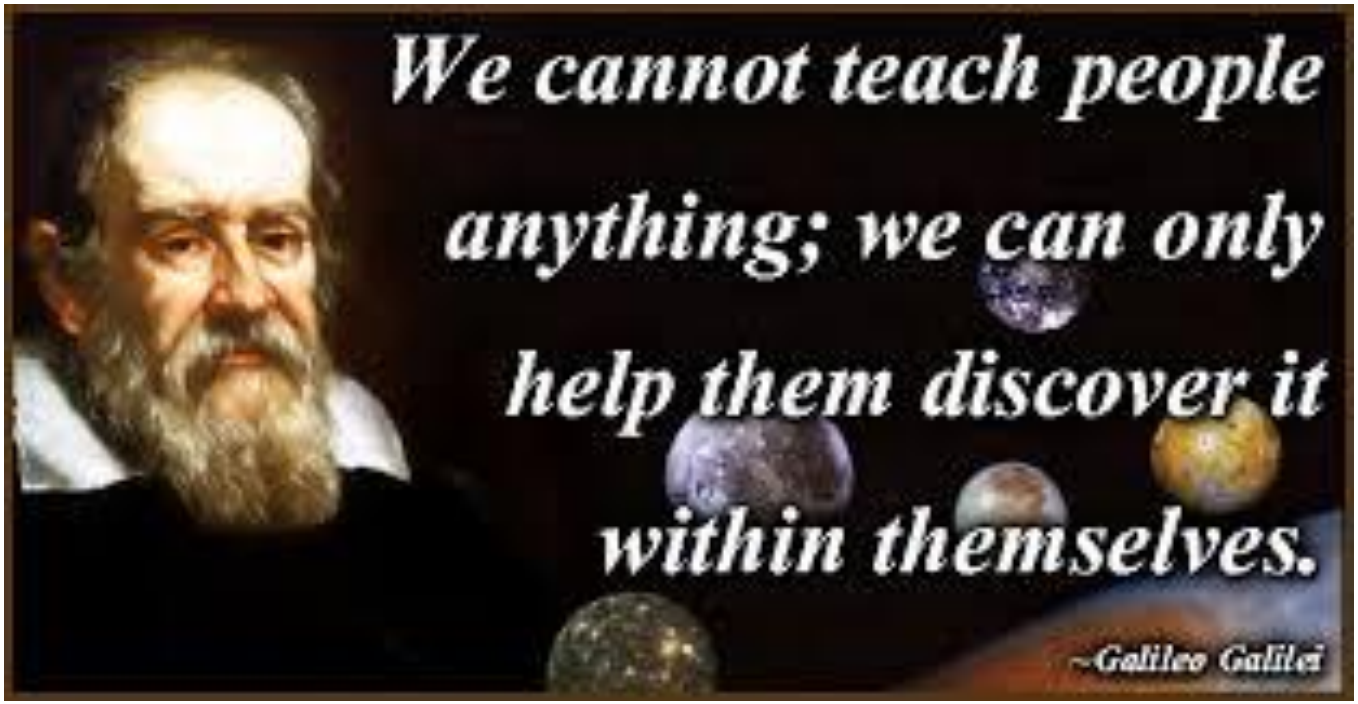
Theory of Relativity

UNIT I
Relativistic Mechanics

Lecture-2


We cannot teach people anything; we can only help them discover it within themselves.
~Galileo Galilei

Failure of Galilean transformation

- The laws of mechanics remain invariant under the Galilean relativity but the laws of electrodynamics do not preserve their form under this theory.

- The failure of the Galilean theory of relativity in the case of electromagnetism suggests some facts which demands some new theory able to readdress the failure of Galilean theory of relativity.

Need of Special Theory of Relativity

- The principle of relativity is meant only for the laws of classical mechanics. It is not applicable for the laws of electromagnetism.

- The principle of relativity is valid for the laws of classical mechanics as well as for the laws of electrodynamics but the laws of electromagnetism need modification in accordance with the Galilean transformations.

- The principle of relativity is valid for the laws of classical mechanics as well as for the laws of electromagnetics but Newton's laws of classical mechanics need modification in accordance with Galilean theory of relativity.
- There is the presence of universal ether medium that provides absolute frame of reference with respect to which all the measurements are performed.

Michelson-Morley

A.A. Michelson
1852 - 1931

E.W. Morley
1838 - 1923

No Ether wind detected: 1907 Nobel Prize

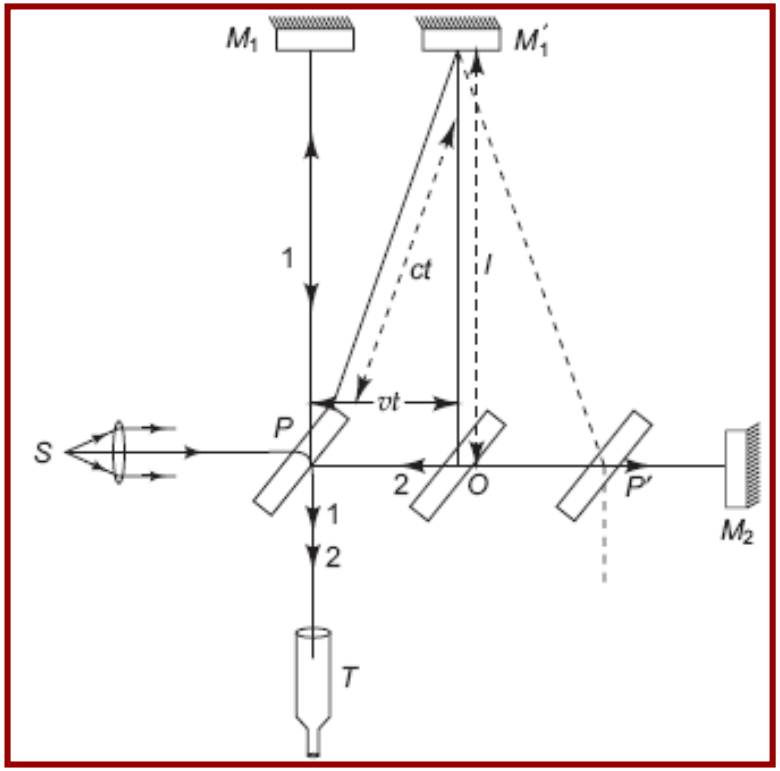
Michelson–Morely Experiment

- It is the experiment performed to test the validity of the presence of ether medium

Results and Discussion

- Initial path difference between ray 1 and 2 is zero.
- Considering the relative motion between earth and ether it is $2lv^2/c^2$
- Corresponding to above path difference the expected fringe shift should be 0.415
- But no fringe shift is observed experimentally.
- This mismatch between experimental and theoretical result is known as negative result of Michelson-Morely experiment.

- **Experimental Setup**



Michelson–Morely Experiment

Now, from the right-angled triangle POM_1', we can write

$$c^2t^2 = v^2t^2 + l^2 \qquad \text{(where } l \text{ is the distance of } M_1 \text{ from } P\text{)}$$

or $$(c^2 - v^2)t^2 = l^2$$

or $$t^2 = \frac{l^2}{\left(c^2 - v^2\right)}$$

or $$t = \frac{l}{\sqrt{c^2 - v^2}}$$

$$t_1 = 2t = \frac{2l}{c\left[1 - \frac{v^2}{c^2}\right]^{1/2}}$$

$$= \frac{2l}{c}\left[1 - \frac{v^2}{c^2}\right]^{-1/2}$$

$$t_1 = \frac{2l}{c}\left[1 + \frac{v^2}{2c^2}\right]$$

Now, the time taken by the ray of light (ray 2) in horizontal direction can be given as

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v}$$

because the relative velocity between the light and the earth will be $c-v$ and $c+v$ when light is approaching mirror M_2 and again returning to P, respectively.

$$t_2 = \frac{2lc}{c^2 - v^2}$$

$$= \frac{2lc}{c^2}\left(1 - \frac{v^2}{c^2}\right)^{-1}$$

or $$t_2 = \frac{2l}{c}\left(1 + \frac{v^2}{c^2}\right)$$ (Neglecting higher-order terms)

Now, the time difference between ray 1 and ray 2 arriving at the glass plate P can be given as

$$\Delta t = t_2 - t_1 = \left[\frac{2l}{c}\left(1 + \frac{v^2}{c^2}\right) - \frac{2l}{c}\left(1 + \frac{v^2}{2c^2}\right)\right]$$

$$= \frac{2l}{c}\left(\frac{v^2}{2c^2}\right)$$

$$= \frac{lv^2}{c^2}$$

Hence, the corresponding effective path difference will be

$$\Delta x = c\,\Delta t = \frac{lv^2}{c^2}$$

When the whole apparatus is rotated by 90°, then the longer path ray takes the place of shorter ray and vice versa. Hence, the path difference will be in opposite sense, i.e., $-lv^2/c^2$, and as a consequence the effective path difference will be doubled,

i.e., $$\Delta x = 2\,\frac{lv^2}{c^2}$$

Due to this path difference, number of fringe shift can be given as

$$\Delta N = \frac{2lv^2/c^2}{\lambda} = \frac{2lv^2}{\lambda c^2}$$

$$\Delta N = \frac{2\times 11\times(3\times 10^4)^2}{(3\times 10^8)^2\times 5.3\times 10^{-7}} = 0.4151$$

But to their utter surprise there was no fringe shift observed experimentally

Explanation of negative result of Michelson Morely Experiment

•***Ether-drag hypothesis:*** According to this hypothesis, the moving earth drags the ether along with it. Therefore, there is no relative motion between the earth and the ether; hence, there will be no question of fringe shift. But this explanation was discarded because it is against the phenomenon of stellar aberration.

• ***Lorentz–FitzGerald contraction hypothesis:*** According to them, all the material bodies are contracted in the direction of motion relative to the stationary ether by a factor of $\sqrt{(1-v^2/c^2)}$. Although this explanation satisfies the experimental result of the Michelson–Morely experiment, even then it was not accepted because the contraction hypothesis was purely mathematical. It was without logic and experimental confirmation.

Explanation of negative result of Michelson Morely Experiment

- ***Constancy of speed of light:*** In 1905, Einstein suggested a true and simple explanation for the negative result of the Michelson–Morely experiment. According to him, the speed of light is constant, and it is independent of the position of the source and the observers.

Einstein's hypotheses:

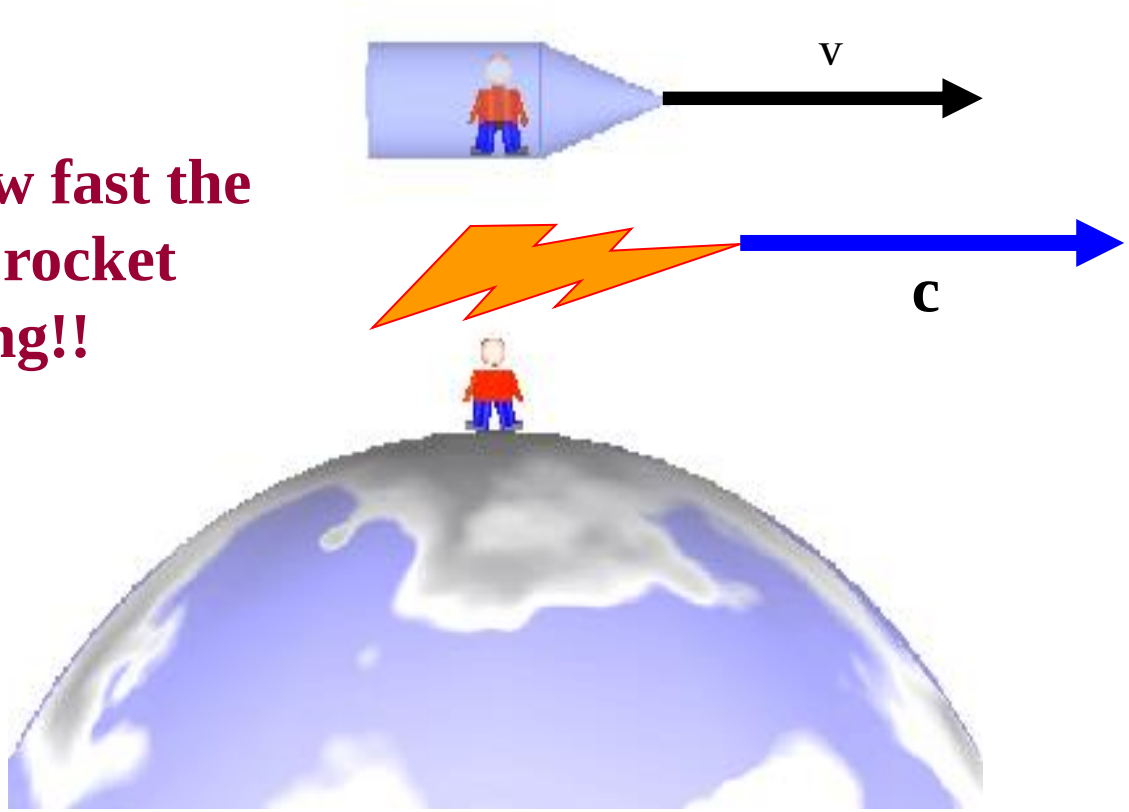
- 1. The laws of nature are equally valid in every inertial reference frame

Including
Maxwell's eqns

2. The speed of light in empty space is same for all inertial observers, regard- less of their velocity or the velocity of the source of light.

All observers see light flashes go by them with the same speed

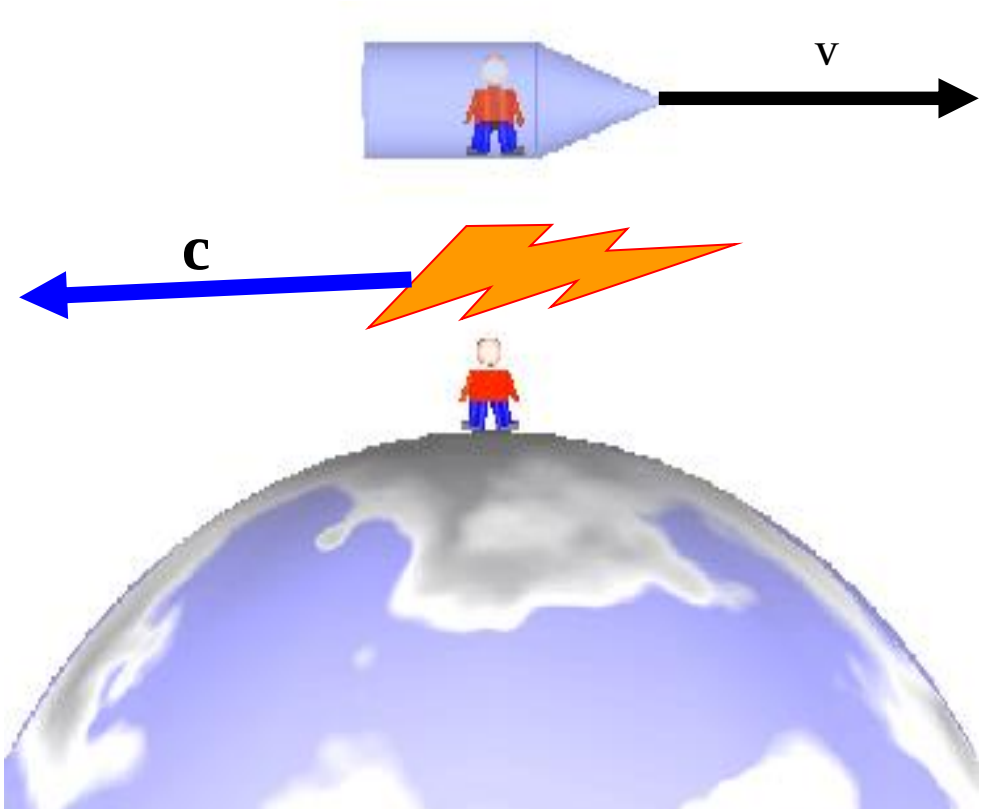
No matter how fast the guy on the rocket is moving!!



Both guys see the light flash travel with velocity = c

Even when the light flash is traveling in an opposite direction

No matter what is the direction guy on the rocket or flash of light is moving!!



Both guys see the light flash travel with velocity = c

Postulates of Special Theory of Relativity

There are two postulates of special theory of relativity

(i) All the laws of physics have the same form in all the inertial frames of reference moving with constant velocity relative to one another.

(ii) The speed of light is constant in all the inertial frames of reference and is independent of the position of the source and the observer. It is equal to the speed of light in free space (vacuum); numerically it is 3×10^8 m/s.

Assignments based on what we learnt in this lecture

- What was the objective of conducting the Michelson Morely Experiment?
- Describe the experimental arrangements of Michelson Morely Experiment.
- Mention the negative results obtained in the Michelson Morely Experiment.
- Explain the negative result of Michelson Morely Experiment.
- Discuss the Lorentz Fitzerald contraction hypothesis.
- Write the postulates of special theory of relativity.

Numerical Problem

Example 1.5

In the Michelson–Morely experiment, what will be the observed relative velocity of earth and the ether, if the two beams have a path of 9 m each and the wavelength of light used is 6000 Å for the expected fringe shift of 0.40 fringe.

Solution

In the Michelson–Morely experiment, the expected fringe shift is given as

$$\Delta n = \frac{2lv^2}{c^2\lambda}$$

or

$$v = c\sqrt{\frac{\lambda \Delta n}{2l}}$$

Given that $l = 9$ m, $\lambda = 6000$ Å, and $\Delta n = 0.40$.

$$v = (3 \times 10^8)\sqrt{\frac{6\times10^{-7}\times 0.4}{2\times 9}}$$

$$= 3.46 \times 10^4 \text{ m/s}$$

Numerical Problem

Example 1.4

What will be the fringe shift in the Michelson–Morely experiment if the effective path length of each path is 9 m and the light used in the experiment is of wavelength 5000 Å. The velocity of earth is 3×10^4 m/s.

Solution

In the Michelson–Morely experiment, the expected fringe shift is given as

$$\Delta n = \frac{2lv^2}{c^2\lambda}$$

or

$$v = c\sqrt{\frac{\lambda \Delta n}{2l}}$$

Given that $l = 9$ m, $\lambda = 6000$ Å, and $\Delta n = 0.40$.

$$v = (3 \times 10^8)\sqrt{\frac{6\times10^{-7}\times 0.4}{2\times 9}}$$

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