MPM: 203 NUCLEAR AND PARTICLE PHYSICS

UNIT –I: Nuclear Stability

Lecture-13

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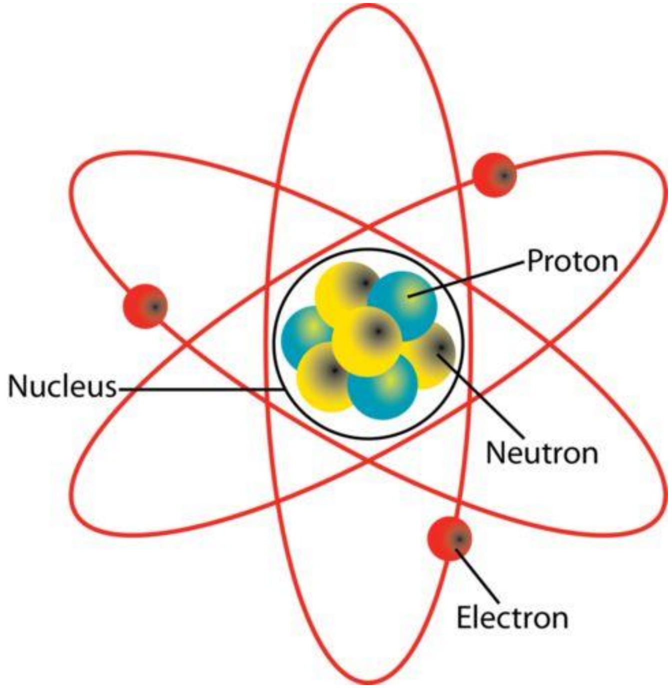

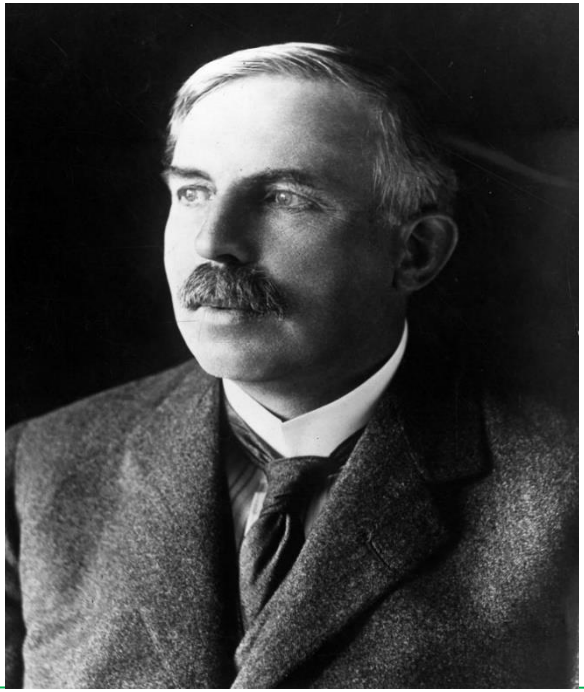

Fermi Theory of Beta Decay

Fermi Theory of β-decay

When a nucleus emits a β-particles, its charge changes by one unit while its mass practically remains unchanged. When ejected β-particle is an electron the number of protons in the nucleus is increased by one and the numbe of neutrons is decreased by one. In positron emission; the reverse process takes place i.e. the number of proton decreases by one number of neutrons increases by one.

β-transformation may then be given as

$$n \longrightarrow p + \beta^- + \bar{\nu}$$

$$p \longrightarrow n + \beta^+ + \nu$$

Fermi Theory of Beta Decay

where ν and $\bar{\nu}$ represent neutrino and antineutrino. In the above equations, neutron is not to be considered as composed of a proton, electron and neutrino but is considered to be transformed into three particles at the instant of β^- emission.

Similarly a proton is transformed into a neutron, positron and neutrino at the time of β^+ emission. The neutron or proton that is transformed, is not a free particle but is bound to the nucleus by the nuclear forces.

The most important thing in the theory of β-decay is to calculate the probability of the above process. This is done as follows:

Fermi Theory of Beta Decay

Actually electron does not exist in the nucleus but it is created at the time of conversion of neutron into proton (at the same time neutrino particle is also emitted. These created particles are represented by the wave functions:

ψ_β and ψ_ν. Let us assume that these are functions for plane waves with momenta p_β and p_ν respectively

$$\psi_\beta = N_\beta e^{i k_\beta \cdot r}$$

$$\psi_\nu = N_\nu e^{i k_\nu \cdot r}$$

Fermi Theory of Beta Decay

where k's are the wave vector and r represents the space co-ordinate and N is a normalization factor.

ψ_β is actually more complicated than shown here because is is affected by the nuclear charge Ze.

The probability of emission can be assumed to depend upon the expectation value for the electron and neutrino to be at the nucleus i.e. on the factor

$$|\psi_\beta(0)|^2 \cdot |\psi_\nu(0)|^2$$

Fermi Theory of Beta Decay

It also depends on other factors, whose nature is uncertain.

One such factor is the square modulus of a matrix element M taken between the initial and final states of the nucleus. This matrix element in its simplest form is

$$M = \int \psi_P^* \psi_N \, d\tau$$

Fermi Theory of Beta Decay

If we consider only one nucleon participates. ψ_N represents the initial state of the nucleon and ψ_P the final state i.e. the proton state. We can also take M to be a vector having x- component given by

$$M_x = \int \psi_P^* \sigma_x \psi_N \, d\tau$$

where σ_x is the x. component of a (relativistic) spin operator. Then

$$|M|^2 = |M_x|^2 + |M_y|^2 + |M_z|^2$$

Fermi Theory of Beta Decay

The choice of M depends upon the selection rules.

The expression for the probability of emission depends also upon a constant factor g^2 which represents the strength of the coupling giving rise to emission and is found to have

$$g = 10^{-48} \text{ to } 10^{-49} \text{ g cm}^5 \text{ sec}^{-2} = 9 \times 10^{-5} \text{ MeV fm}^3$$

Thus the probility of emission per unit time is

$$\frac{2\pi}{\hbar}\left[|\psi_\beta(0)| \cdot |\psi_\nu(0)| \cdot |M| \cdot |g|^2 \frac{dn}{dE}\right] \quad — (i)$$

Fermi Theory of Beta Decay

where $\frac{dn}{dE}$ represents the energy density of the final states and o represents to the location of the nucleus.

Let Ω be the volume of a big box in which we enclose the system for normalization purposes. then

$$\int_{\Omega} \psi^* \psi \, d\tau = 1 \quad \therefore \; N = \frac{1}{\sqrt{\Omega}}$$

Fermi Theory of Beta Decay

Therefore $\psi_\beta = \frac{1}{\Omega^{1/2}} e^{i k_\beta \cdot r}$

$$\psi_\nu = \frac{1}{\Omega^{1/2}} e^{i k_\nu \cdot r}$$

where $k_\beta = \frac{p_\beta}{\hbar}$, $k_\nu = \frac{p_\nu}{\hbar}$ when nucleus is at $r = 0$

$$\psi_\nu(0) = \frac{1}{\sqrt{\Omega}} = \psi_\beta(0)$$

The number of plane wave states having magnitude of momentum between p and $p+dp$ with the particle anywhere in the volume Ω is given by

$$\frac{p^2 \, dp \, \Omega}{2\pi^2 \hbar^3}$$

Fermi Theory of Beta Decay

Now

$$dn = \frac{p_\beta^2\, dp_\beta}{2\pi^2 \hbar^3} \times \frac{p_\nu^2\, dp_\nu}{2\pi^2 \hbar^3}\, \Omega^2$$

$$dn = \Omega^2 \frac{p_\beta^2\, p_\nu^2}{4\pi^4 \hbar^6}\, dp_\beta \cdot dp_\nu$$

We know the relation $E = E_\nu + E_\beta = cp_\nu + E_\beta$

$$dE = c \cdot dp_\nu$$

$$dE \cdot dp_\beta = c \cdot dp_\beta \cdot dp_\nu$$

Thus

$$dn = \Omega^2 \frac{p_\beta^2\, p_\nu^2}{4\pi^4 \hbar^6} \frac{1}{c}\, dE\, dp_\beta$$

or

$$\frac{dn}{dE} = \Omega^2 \frac{p_\beta^2\, p_\nu^2}{4\pi^2 \hbar^6} \frac{1}{c} \cdot dp_\beta \quad \text{— (2)}$$

Fermi Theory of Beta Decay

using above equation in eq. ① we get the proba-
-bility of emission per unit time $P(p_\nu, p_\beta)dp_\beta$ is

$$P(p_\nu, p_\beta)\, dp_\beta = \frac{2\pi}{\hbar}\left(\frac{1}{\Omega}|M|g\right)^2 \frac{\Omega^2 p_\beta^2 p_\nu^2 dp_\beta}{4\pi^2\hbar^6 c}$$

Using the relation

$$p_\nu c = E_\nu = E_\beta^{max} - E_\beta$$

to eliminate p_ν and replacing p_β by simply p, we get

Fermi Theory of Beta Decay

$$P(p)\,dp = \frac{g^2 |M|^2}{2\pi^3 \hbar^7 c^3} \left(E_\beta^{max} - E_\beta\right)^2 p^2\, dp$$

using the relation

$$E_\beta^{max} = \sqrt{m^2c^4 + c^2 p_{max}^2}$$

we get

$$\boxed{P(p)\,dp = \frac{g^2 |M|^2}{2\pi^3 c^3 \hbar^7} \left(\sqrt{m^2c^4 + c^2 p_{max}^2} - \sqrt{m^2c^4 + c^2 p^2}\right)^2 p^2\, dp}$$