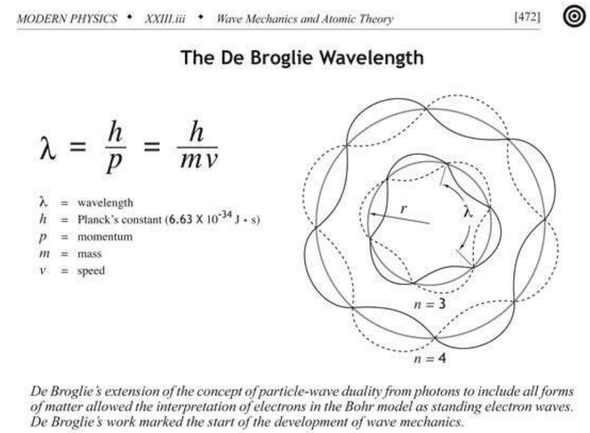
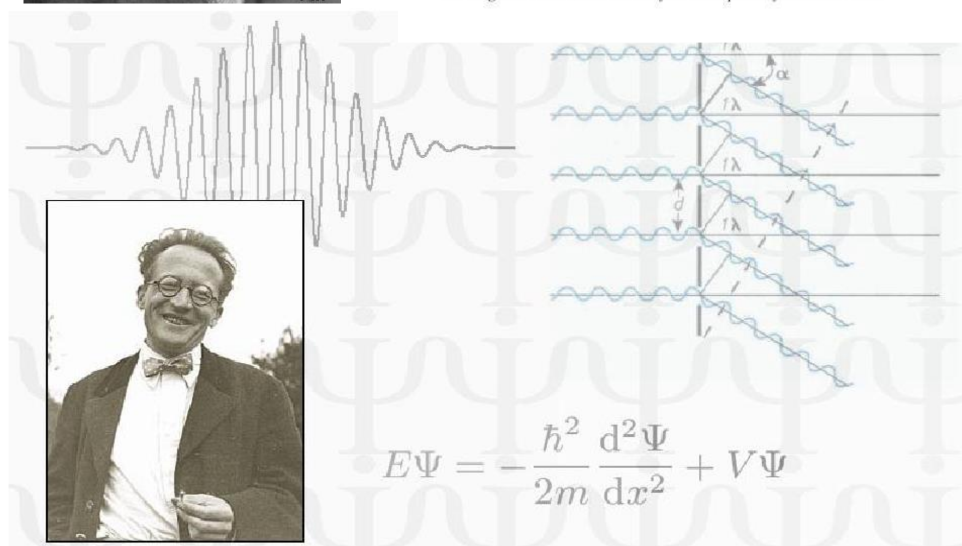
QUANTUM MECHANICS

UNIT II
Quantum Mechanics
Lecture-2





NATURE OF ELECTRON

- Before 1924, the electron was exclusively regarded as a particle but after the suggestion of de Broglie, the electron was given the wave–particle duality.

- There are many experimental evidences which prove that the electron is a particle, as it has a definite mass, charge, energy, and momentum.

- Additionally, the impact of the electron on the screen of zinc sulphide proves its identity as a particle.

NATURE OF ELECTRON

- There are some experimental evidences which prove that the electron has a wave nature as well, i.e., a wave is associated with the electron during its motion. The following are
- Some experiments which support the wave nature of electrons:

- **Davisson and Germer diffraction experiment**
- **G.P. Thomson experiment**

DAVISSON AND GERMER EXPERIMENT FOR MATTER-WAVES

- ***Principle:*** If an electron is accelerated through a potential difference of V volts, then the electron acquires the kinetic energy equivalent to qV joule, where q is the charge on the electron (in coulomb). The total energy E of the electron becomes :

$$E = m_0c^2 + qV$$

where m_0c^2 is the rest-mass energy of the electron.

But from the special theory of relativity, we have

$$E = \sqrt{m_0^2c^4 + p^2c^2}$$

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$$E = \sqrt{m_0^2c^4 + p^2c^2}$$

$$m_0{}^2c^4 + p^2c^2 = (m_0c^2 + qV)^2$$

$$p^2c^2 = 2m_0c^2\, qV + (qV)^2$$

$$p = \sqrt{\left\{2m_0qV\left(1 + \frac{qV}{2m_0c^2}\right)\right\}}$$

DAVISSON AND GERMER EXPERIMENT FOR MATTER-WAVES

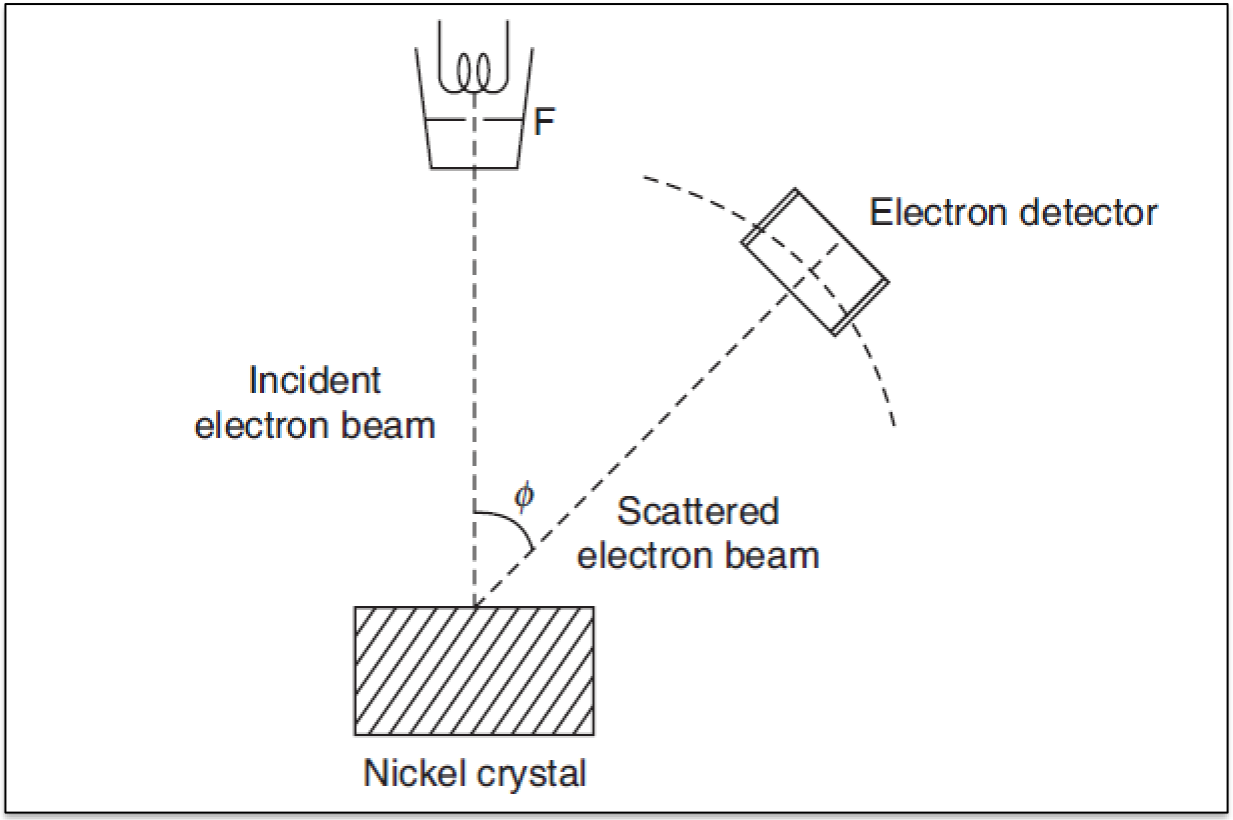
The de Broglie wavelength corresponding to such an electron is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{\{2m_0 qV(1 + qV / 2m_0c^2)\}}}$$

With above expressions de Broglie hypothesis gives us

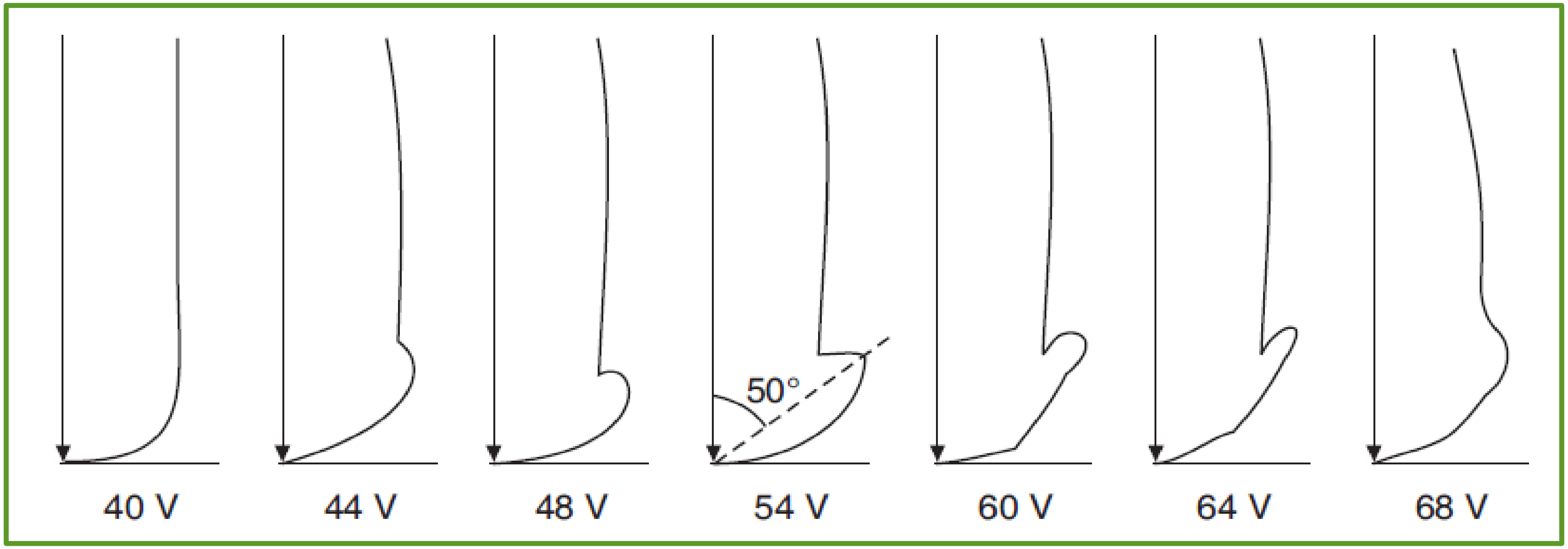
$$\boxed{\lambda = \frac{12.27}{\sqrt{V}}\ \text{Å}}$$

EXPERIMENTAL SET UP



Results and discussion:

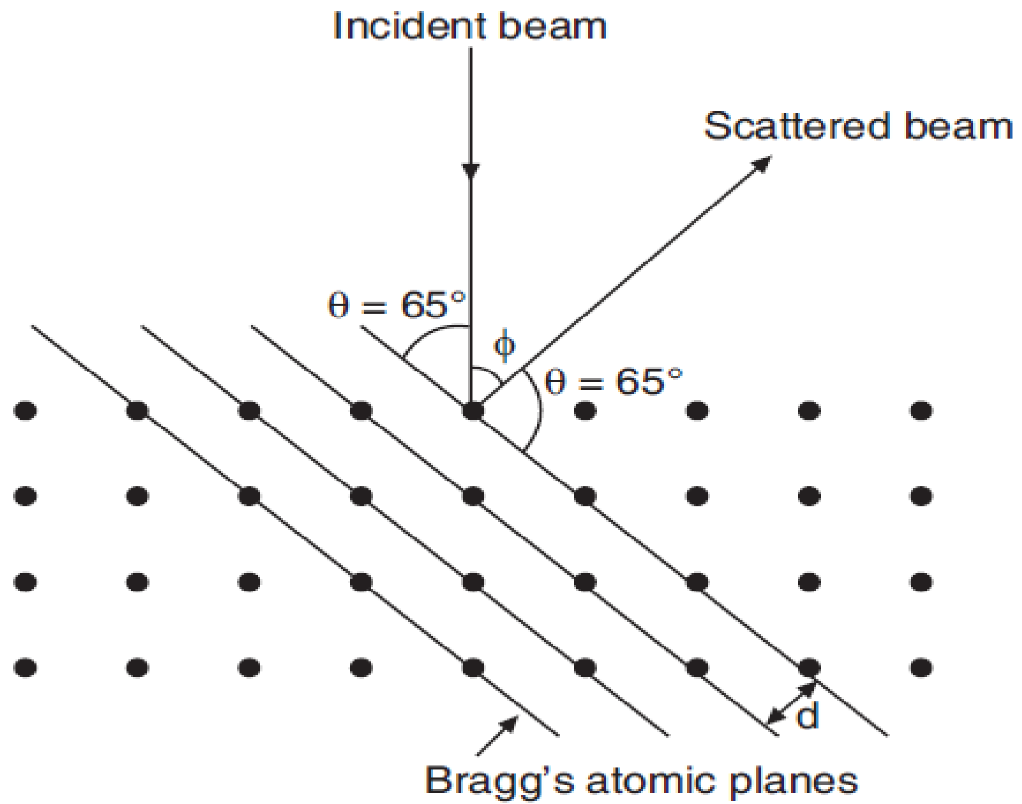
- *The experimental results are discussed on the basis of different curves obtained between scattering angle ϕ and the intensity of scattered beam of electrons, corresponding to different accelerating voltages V. It is observed that intensity is maximum at 54 volt corresponding to the scattering angle of 50^0 .*



Curves showing the variation of intensity with scattering angle at different accelerating potentials

Bragg's atomic planes

In order to test the theoretical result experimentally we use Bragg's law



Scattering of electron wave from Bragg's atomic planes in nickel crystal

DAVISSON AND GERMER EXPERIMENT

- Using Bragg's law we get $\theta = 65^0$. Interatomic spacing d=0.91 A^0

- If λ is the wavelength of the de Broglie wave associated with the electron, then according to Bragg's law, we can write

$$\mathbf{2d \sin \theta = n\lambda}$$

- where d is the inter-atomic spacing and n is the order of diffraction. For the nickel crystal d = 0.91 Å,

- For first-order (n = 1) diffraction maxima,
  $\lambda = 2 \times 0.91$ Å $\times \sin 65^\circ = 1.65$ Å

DAVISSON AND GERMER EXPERIMENT

- de Broglie wavelength can also be calculated from Eq. (22.11), for V = 54 V as given below:
- $\lambda = \frac{12.27}{\sqrt{54}} = = 1.67$ Å
- This value of wavelength is very close to the experimentally obtained value of λ.
- Hence, it can be concluded that a wave is associated with a particle during its motion.
- Thus the de Broglie hypothesis of matter-waves is experimentally proved.

Example-1

Calculate the de Broglie wavelength associated with a proton moving with a velocity equal to (1/20)th of the velocity of light.

Solution

de Broglie wavelength associated with a proton is given as

$$\lambda = \frac{h}{m\upsilon}$$

It is given that $h = 6.625 \times 10^{-34}$ Js, $m_p = 1.67 \times 10^{-27}$ kg, and $\upsilon = c/20 = 3 \times 10^8$ m/s

Now

$$\lambda = \frac{6.625\times10^{-34}\times20}{1.67\times10^{-27}\times3\times10^{8}}$$

$$= 6.645 \times 10^{-14} \text{ m}$$

Find the de Broglie wavelength of a neutron of energy 12.8 MeV (given that, $h = 6.625 \times 10^{-34}$ Js, mass of neutron (m_n) = 1.675×10^{-27} kg, and 1 eV = 1.6×10^{-19} J).

Solution

The rest-mass energy of neutron is given as

$$m_0c^2 = 1.675 \times 10^{-27} \times (3 \times 10^{-8})^2$$

$$= 1.5075 \times 10^{-10} \text{ J}$$

or

$$= \frac{1.507\times10^{-10}}{1.6\times10^{-19}} = 941.88 \times 10^6 \text{ eV}$$

$$= 941.88 \text{ MeV}$$

The given energy 12.8 MeV is very less compared to the rest-mass energy of neutron, therefore, relativistic consideration in this case is not applicable. Now, the de Broglie wavelength of the neutron is given as

$$\lambda = \frac{h}{\sqrt{2mE_k}}$$

where $E = 12.8 \text{ MeV} = 1.28 \times 10^6 \times (1.6 \times 10^{-19})$ J

Now

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times1.675\times10^{-27}\times12.8\times10^6\times1.6\times10^{-19}}}$$

$$= \frac{6.62\times10^{-34}}{8.28\times10^{-20}}$$

$$= 8.04 \times 10^{-15} \text{ m}$$

$$= 8.04 \times 10^{-5} \text{ Å}$$

Example-3

A particle of charge q and mass m is accelerated from rest through a potential difference V. Find its de Broglie wavelength. Calculate the wavelength (λ), if the particle is an electron and V = 50 V.

Solution

Let us consider an e The kinetic energy of the electr

$$= \frac{h}{\sqrt{2eVm_0}}$$

$\frac{1}{2}$ where h is Planck's constant (6.625×10^{-34} Js).

Hence, the veloc

Now, by putting the values of constants, we get

v :

$$\lambda = \frac{6.625\times10^{-34}}{\sqrt{[2\times1.632\times10^{-19}\times V\times9.1\times10^{-31}]}}$$

Thus, the de Bro

$$= \frac{12.26}{\sqrt{V}}\ \text{Å}$$

λ :

For the electron, it is given that $V = 50$ V. Hence,

$$\lambda = \frac{12.27}{\sqrt{50}}\ \text{Å}$$

$$= 1.735\ \text{Å}$$

Example-4

Calculate the de Broglie wavelength of an electron having its energy V electron volt.

Solution

We know that the de Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

For electron $m = 9.1 \times 10^{-31}$ kg

Given that $E = V \times 1.6 \times 10^{-19}$ J

and $h = 6.62 \times 10^{-34}$ Js

Now,
$$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$$

$$\lambda = \frac{12.27}{\sqrt{V}}\ \text{Å}$$

For an electron and a photon each having a wavelength of 1.0 Å, compare their

(i) momentum,

(ii) total energy, and

(iii) ratio of kinetic energy.

(Given that $h = 6.63 \times 10^{-34}$ Js, rest mass of electron (m_0) is 9.1×10^{-31} kg, $c = 3.0 \times 10^{8}$ m/s, and 1 eV = 1.6×10^{-19} J)

Solution

(i) The de Broglie wavelength associated with a particle during its motion is given as

$$\lambda = \frac{h}{p}$$

or $$p = \frac{h}{\lambda}$$

Now, the momentum of the electron is given as

$$p_e = \frac{6.63\times10^{-34}}{1\times10^{-10}} = 6.63 \times 10^{-24} \text{ kgm/s}$$

For the photon,

$$p_p = \frac{h}{\lambda} = \frac{h}{c/\nu} = \frac{h\nu}{c}$$

$$p_p = 6.63 \times 10^{-24} \text{ kgm/s}$$

Thus, for the same wavelength, both the photon and the electron have the same momentum.

(ii) The total energy of the electron can be given as

$$\text{Total energy } (E) \text{ of electron} = \text{kinetic energy} + \text{rest-mass energy}$$

$$= \frac{p_e^2}{2m_e} + m_0c^2$$

$$= \frac{(6.63\times10^{-24})^2}{2\times9.1\times10^{-31}} + \frac{9.1\times10^{-31}\times(3\times10^8)^2}{1.6\times10^{-19}}$$

$$= 2.42 \times 10^{-17} + \frac{8.19\times10^{-14}}{1.6\times10^{-19}}$$

$$E = 0.512 \times 10^6 \text{ eV}$$

(because the first term is negligible)

Since the first term in the above equation is negligible in comparison to the second one, so

$$E = 0.512 \times 10^6 \text{ eV}$$
$$= 0.51 \text{ MeV}$$

The total energy of the photon (E) is given as

$$E = h\nu = \frac{hc}{\lambda} = p_c$$
$$= 6.63 \times 10^{-24} \times 3 \times 10^8$$
$$= 1.989 \times 10^{-15} \text{ J}$$
$$= \frac{1.989\times10^{-15}}{1.6\times10^{-19}} = 1.24 \times 10^4 \text{ eV}$$
$$= 12.4 \text{ keV}$$

Since the rest-mass energy of the photon is zero, the total energy of the photon will be equal to its kinetic energy. Hence, the kinetic energy of the photon will be 12.4 keV.

(iii) The kinetic energy of the electron (K_e) is given as

$$K_e = 2.42 \times 10^{-17} \text{ J}$$
$$= \frac{2.42\times10^{-17}}{1.6\times10^{-19}} \text{ eV}$$
$$= 1.51 \times 10^2 \text{ eV}$$
$$= 0.151 \text{ keV}$$

The kinetic energy of the photon (K_p) is 12.4 keV. That is,

$$K_p = 12.4 \text{ keV}$$

Now,

$$\frac{K_e}{K_p} = \frac{0.151 \text{ keV}}{12.4 \text{ keV}}$$
$$= 1.23 \times 10^{-2}$$

Assignment Based on this Lecture

- How do you conclude that electron have dual nature.
- Describe the suitable experiment which confirm that electron have wave nature.
- Discuss the basic principle and experimental arrangement of Davission Germer Experiment.
- Discuss the result of Davission Germer Experiment.

Prove that the de Broglie wavelength of a particle of rest mass m_0 and charge q, accelerated by a potential difference V is given by

$$\lambda = \frac{h}{\sqrt{2m_0qV(1+qV/2m_0c^2)}}$$