Control Systems

Subject Code: BEC-26 Third Year ECE

Unit-I

Shadab A. Siddique
Assistant Professor

Maj. G. S. Tripathi
Associate Professor

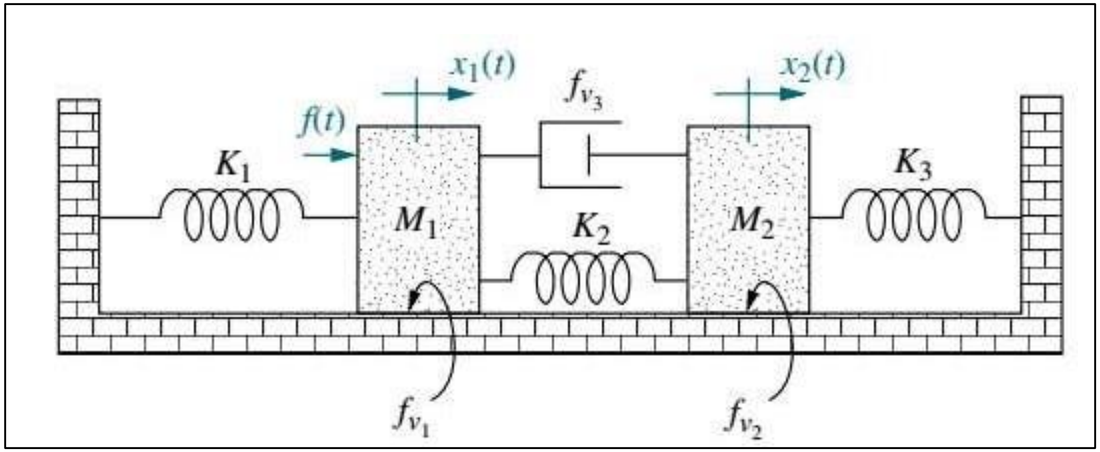
Lecture 9

Department of Electronics & Communication Engineering,
Madan Mohan Malaviya University of Technology, Gorakhpur

Force Analysis

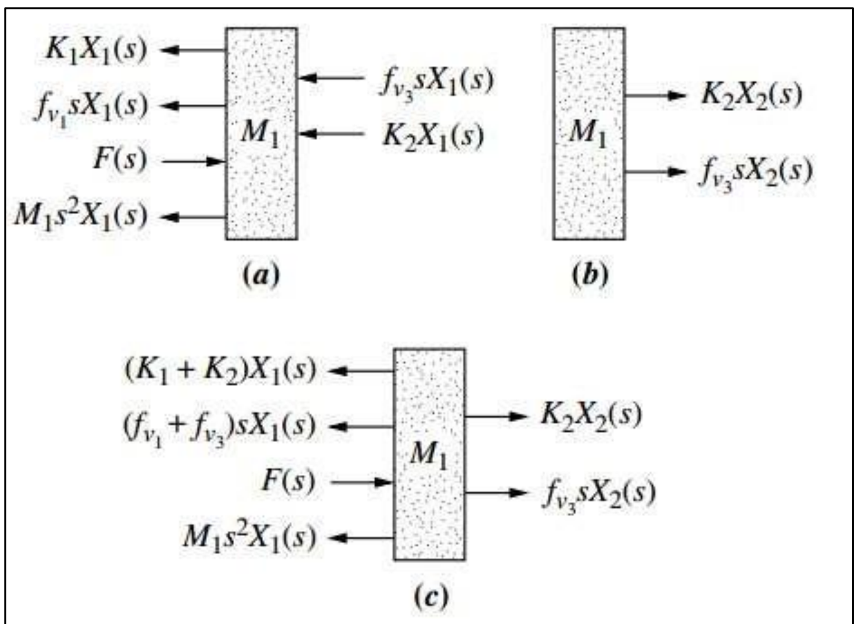
Forces on M_1

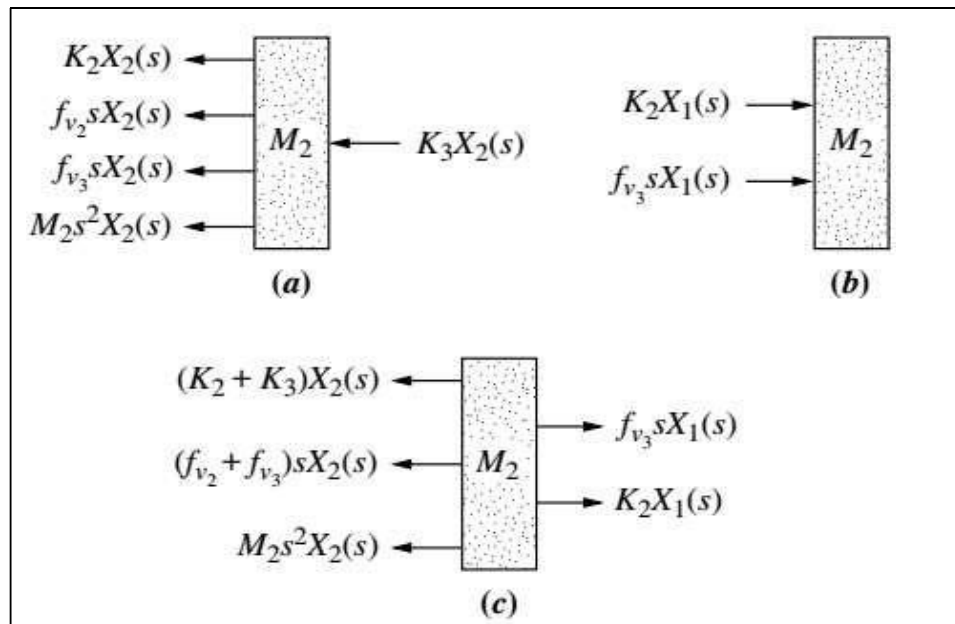
a. Hold M_2 still, move M_1 to right
b. Hold M_1 still, move M_2 to right
c. combined



Forces on M_2

a. Hold M_1 still, move M_2 to right
b. Hold M_2 still, move M_1 to right
c. combined





Use Analogy

$$\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } x_1\end{bmatrix} X_1(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_1 \text{ and } x_2\end{bmatrix} X_2(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied forces}\\ \text{at } x_1\end{bmatrix}$$

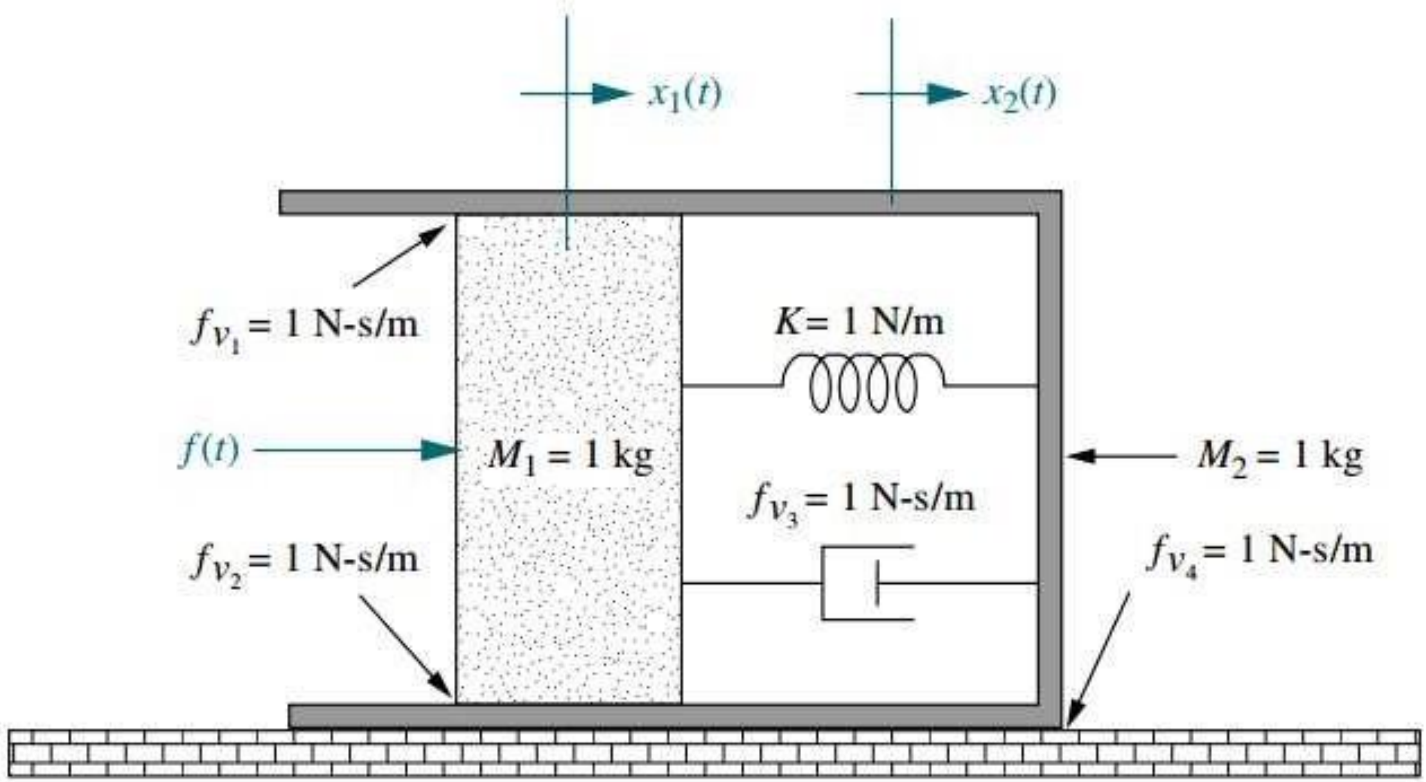
$$-\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_1 \text{ and } x_2\end{bmatrix} X_1(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } x_2\end{bmatrix} X_2(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied forces}\\ \text{at } x_2\end{bmatrix}$$

Loop 1 $[(M_1 s^2 + (f_{v_1} + f_{v_3})s + (K_1+K_2)]\, X_1(s) - (K_2+f_{v_3}s)X_2(s) = F(s)$

Loop 2 $-(K_2+f_{v_3}s)X_1(s) + [(M_2 s^2 + (f_{v_2} + f_{v_3})s + (K_1+K_2)]X_2(s) = 0$

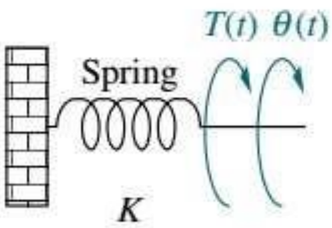
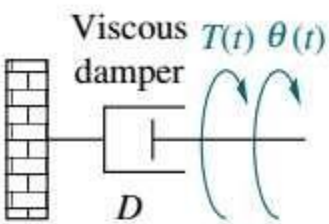
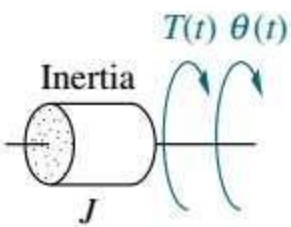
Homework

Find the transfer function, $G(s) = X_2(s)/F(s)$



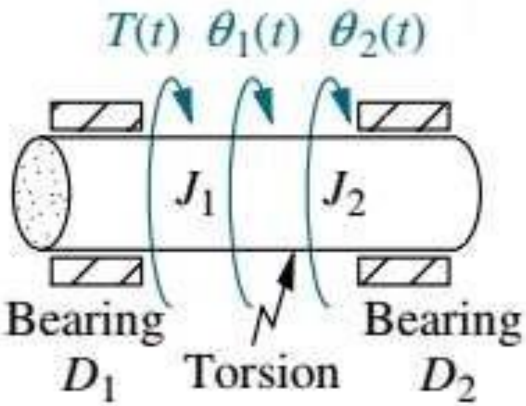
Mechanical Systems (Rotational)

- Torque replaces force; angular displacement replaces translational displacement.

Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\frac{d\theta(t)}{dt}$	Ds
	$T(t) = J\frac{d\omega(t)}{dt}$	$T(t) = J\frac{d^2\theta(t)}{dt^2}$	Js^2

Example#1:

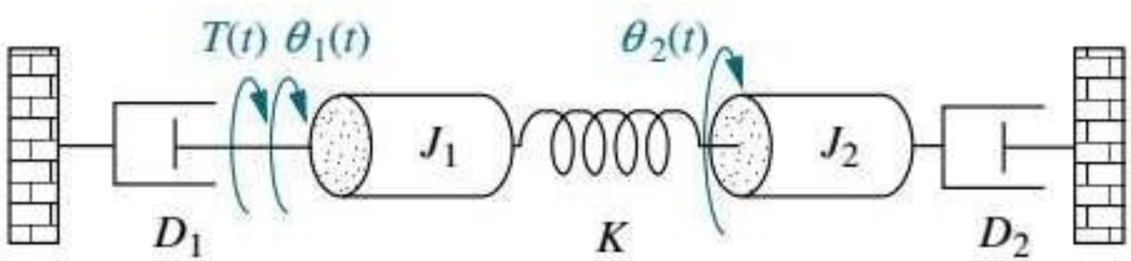
PROBLEM: Find the transfer function, $\theta_2(s)/T(s)$, for the rotational system shown



Physical system

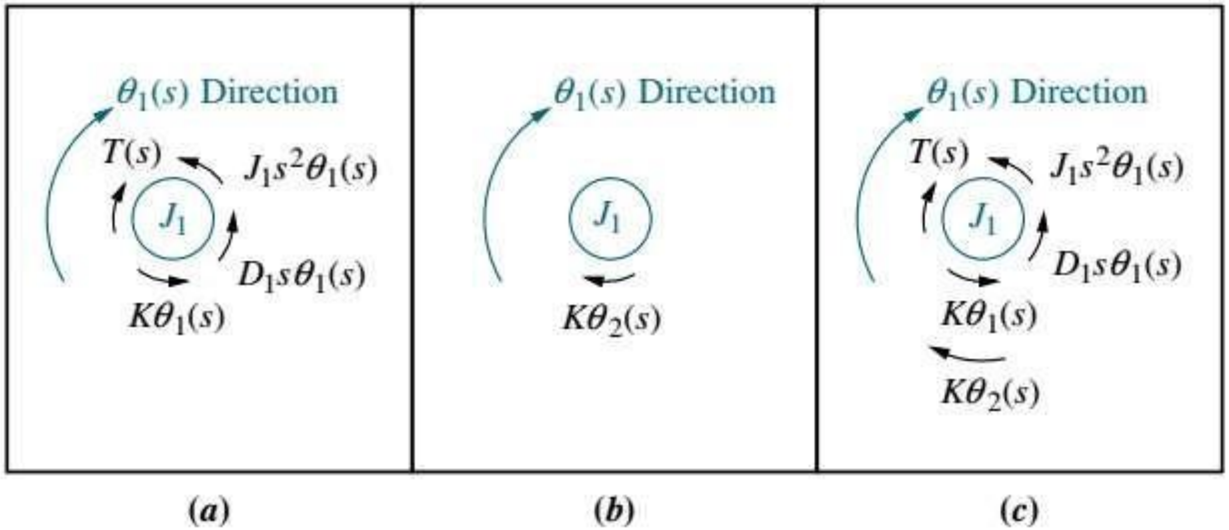
The rod is supported by bearings at either end and is undergoing torsion.

A torque is applied at the left, and the displacement is measured at the right.



Schematic of the system

Loop 1



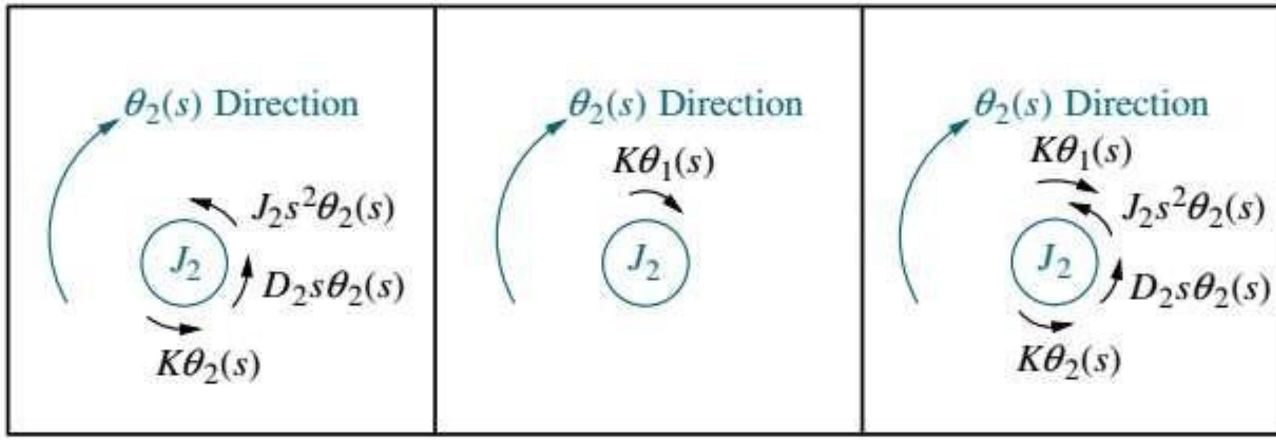
a. Torques on J_1 due only to the motion of J_1; **b.** torques on J_1 due only to the motion of J_2; **c.** final free-body diagram for J_1

$$\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } \theta_1\end{bmatrix}\theta_1(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ \theta_1 \text{ and } \theta_2\end{bmatrix}\theta_2(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied torques}\\ \text{at } \theta_1\end{bmatrix}$$

$$(J_1s^2 + D_1s + K)\theta_1(s) \quad - K\theta_2(s) = T(s)$$

Loop 2

a. Torques on J_2 due only to the motion of J_2; **b.** torques on J_2 due only to the motion of J_1; **c.** final free-body diagram for J_2



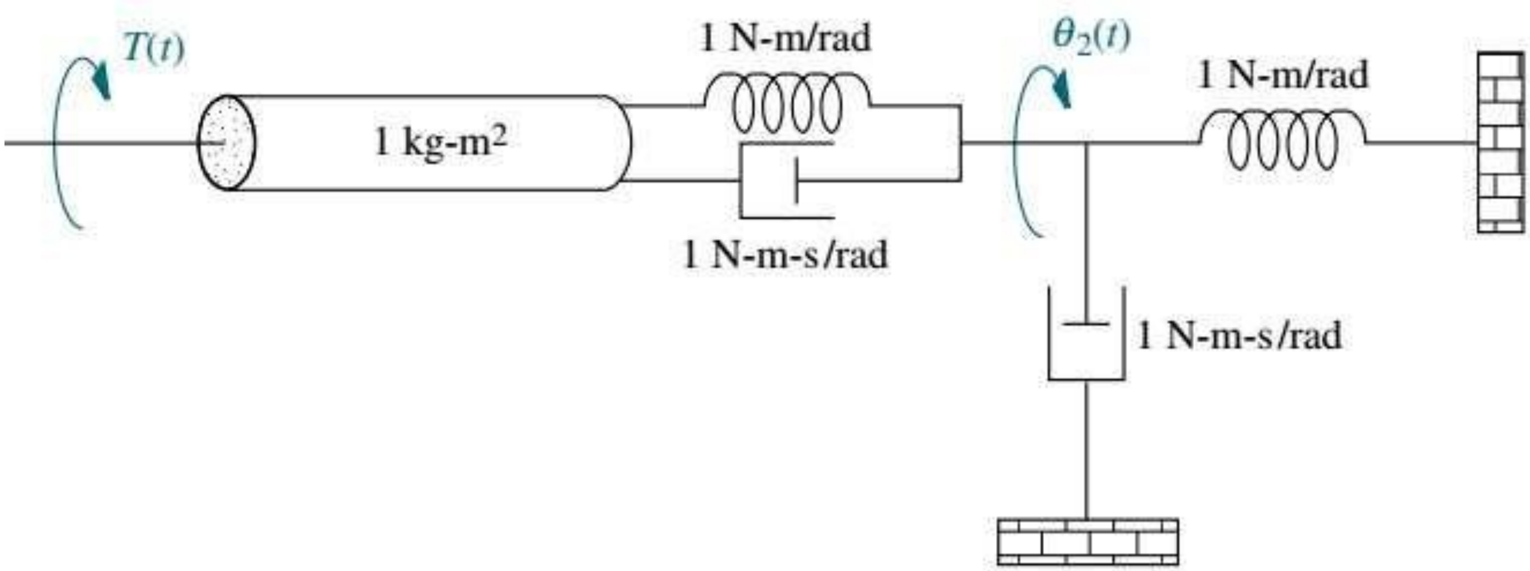
(a) (b) (c)

$$-\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ \theta_1 \text{ and } \theta_2\end{bmatrix}\theta_1(s)+\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } \theta_2\end{bmatrix}\theta_2(s)=\begin{bmatrix}\text{Sum of}\\ \text{applied torques}\\ \text{at } \theta_2\end{bmatrix}$$

$$-K\theta_1(s)+(J_2s^2+D_2s+K)\theta_2(s)=0$$

Homework

PROBLEM: Find the transfer function, $G(s) = \theta_2(s)/T(s)$.



UNIT- I

- Introduction to Control system
  - Control System – Definition and Practical Examples
  - Basic Components of a Control System
- Feedback Control Systems:
  - Feedback and its Effect
  - Types of Feedback Control Systems
- Block Diagrams:
  - Representation and reduction
  - Signal Flow Graphs
- Modeling of Physical Systems:
  - Electrical Networks and Mechanical Systems
  - Force-Voltage Analogy
  - Force-Current Analogy

Electrical Analogies of Mechanical Systems

Two systems are said to be **analogous** to each other if the following two conditions are satisfied.

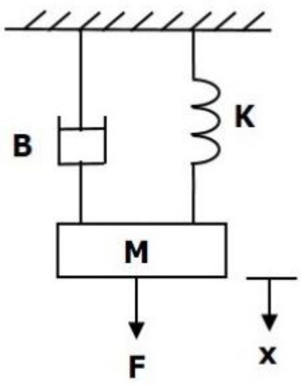
- The two systems are physically different
- Differential equation modelling of these two systems are same

Electrical systems and mechanical systems are two physically different systems. There are two types of electrical analogies of translational mechanical systems. Those are force voltage analogy and force current analogy.

Force to Voltage Analogy:

In force voltage analogy, the mathematical equations of **translational mechanical system** are compared with mesh equations of the electrical system.

Consider the following translational mechanical system as shown in the following figure.



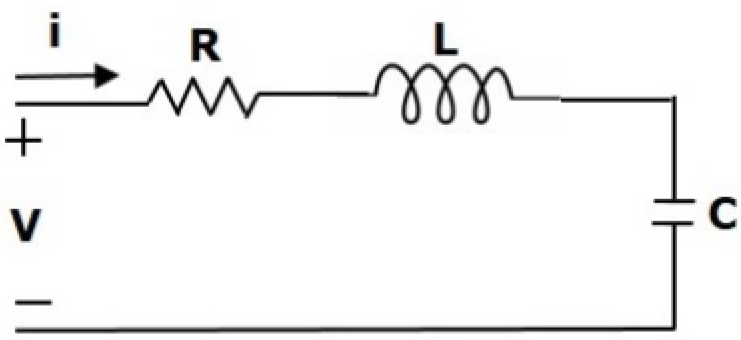
The **force balanced equation** for this system is

$$F = F_m + F_b + F_k$$

$$\Rightarrow F = M\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + B\frac{\mathrm{d}x}{\mathrm{d}t} + Kx \qquad \textbf{(Equation 1)}$$

Consider the following electrical system as shown in the following figure. This circuit consists of a resistor, an inductor and a capacitor. All these electrical elements are connected in a series.

The input voltage applied to this circuit is V volts and the current flowing through the circuit is i Amps.



Mesh equation for this circuit is

$$V = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{1}{c}\int i dt$$ **(Equation 2)**

Substitute, $i = \frac{\mathrm{d}q}{\mathrm{d}t}$ in Equation 2.

$$V = R\frac{\mathrm{d}q}{\mathrm{d}t} + L\frac{\mathrm{d}^2q}{\mathrm{d}t^2} + \frac{q}{C}$$

$$\Rightarrow V = L\frac{\mathrm{d}^2q}{\mathrm{d}t^2} + R\frac{\mathrm{d}q}{\mathrm{d}t} + \left(\frac{1}{c}\right)q$$ **(Equation 3)**

By comparing Equation 1 and Equation 3, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

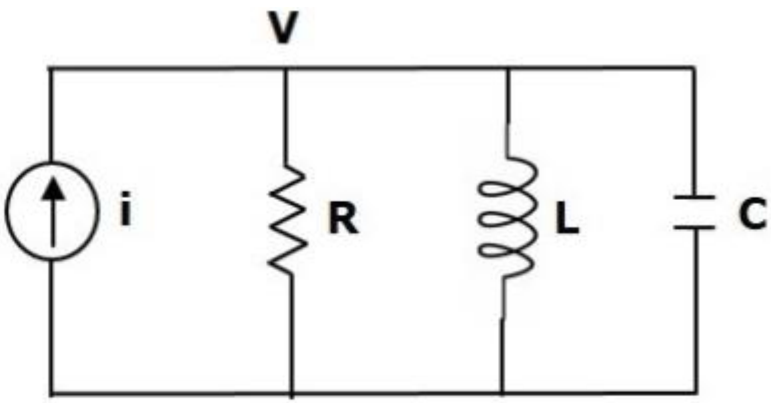
Translational Mechanical System	Electrical System
Force(F)	Voltage(V)
Mass(M)	Inductance(L)
Frictional Coefficient(B)	Resistance(R)
Spring Constant(K)	Reciprocal of Capacitance $(\frac{1}{c})$
Displacement(x)	Charge(q)
Velocity(v)	Current(i)

Similarly, there is torque voltage analogy for rotational mechanical systems. Let us now discuss about this analogy.

Force to Current Analogy:

In force current analogy, the mathematical equations of the **translational mechanical system** are compared with the nodal equations of the electrical system.

Consider the following electrical system as shown in the following figure. This circuit consists of current source, resistor, inductor and capacitor. All these electrical elements are connected in parallel.



The nodal equation is

$$i = \frac{V}{R} + \frac{1}{L}\int V dt + C\frac{\mathrm{d}V}{\mathrm{d}t}$$ **(Equation 5)**

Substitute, $V = \frac{\mathrm{d}\Psi}{\mathrm{d}t}$ in Equation 5.

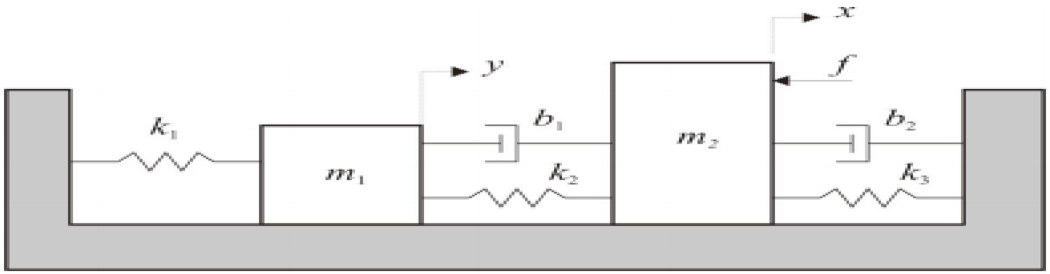
$$i = \frac{1}{R}\frac{\mathrm{d}\Psi}{\mathrm{d}t} + \left(\frac{1}{L}\right)\Psi + C\frac{\mathrm{d}^2\Psi}{\mathrm{d}t^2}$$

By comparing Equation 1 and Equation 6, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Translational Mechanical System	Electrical System
Force(F)	Current(i)
Mass(M)	Capacitance(C)
Frictional coefficient(B)	Reciprocal of Resistance $(\frac{1}{R})$
Spring constant(K)	Reciprocal of Inductance $(\frac{1}{L})$
Displacement(x)	Magnetic Flux(ψ)
Velocity(v)	Voltage(V)

Similarly, there is a torque current analogy for rotational mechanical systems. Let us now discuss this analogy.

1.

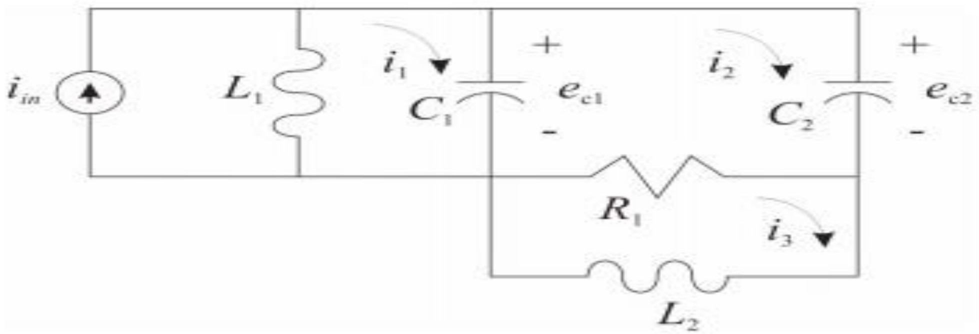


No sliding friction.

a. Find the analogous electrical system using the force-voltage analogy. Indicate the values of all components in the electrical system (resistors, capacitors, inductors, etc.). Show $\dot{x}$ and $\dot{y}$ on your electrical system. Explain the steps you use to find your electrical system.

b. Find the analogous electrical system using the force-current analogy. Indicate the values of all components in the electrical system (resistors, capacitors, inductors, etc.). Show $\dot{x}$ and $\dot{y}$ on your electrical system. Explain the steps you use to find your electrical system.

2.



Find the analogous mechanical system using the **force-current** analogy. Indicate the values of all components in the mechanical system (springs, masses, dampers, etc.). Show e_{c1} and e_{c2} on your mechanical system. Explain the steps you use to find your mechanical system.

UNIT-I

The End

Thank You