Control Systems

Subject Code: BEC-26 Third Year ECE

Unit-III

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Lecture 1

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UNIT - III

Time response of continuous data systems, Different test Signals for the time response, Unit step response and Time-Domain Specifications, Time response of a first-order and second order systems for different test signals, Steady State Error and Error constants, Sensitivity, Control Actions: Proportional, Derivative, Integral and PID control.

- ✓ Time Response of a Continuous data systems/ Time Domain Analysis
- ✓ Transient and Steady State Response
- ✓ Standard Test Signal : Step, Ramp, Parabolic and Impulse, Need, Significance and corresponding Laplace Representation
- ✓ Poles and Zeros : Definition, S-plane representation
- ✓ First Order Control System : Analysis for step Input, Concept of Time Constant
- ✓ Second Order Control System : Analysis for step input, Concept, Definition and effect of damping
- ✓ Time Response Specifications (no derivations)
- ✓ Tp, Ts, Tr, Td, Mp, ess – problems on time response specifications
- ✓ Steady State Analysis – Type 0, 1, 2 system,
- ✓ steady state error constants, problems
- ✓ Control Actions: Proportional, Derivative, Integral, PI, PD and PID control action

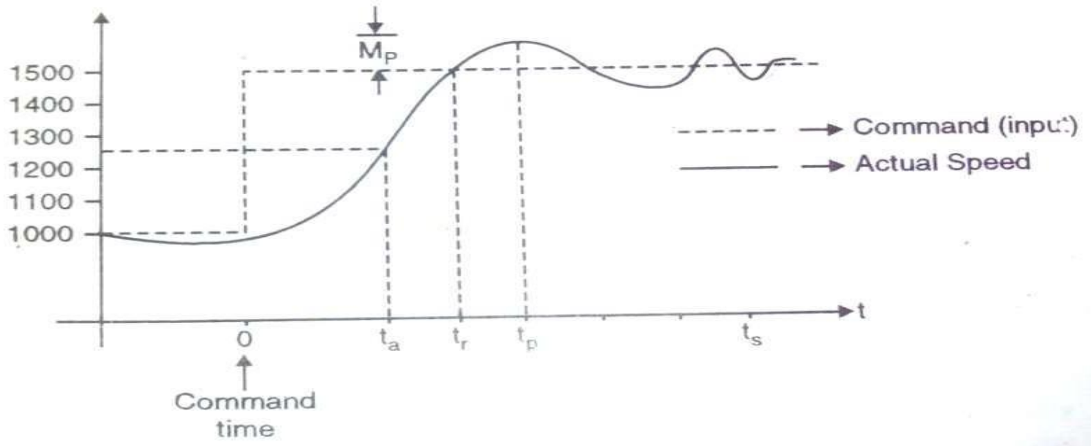
Time Response

- **In time domain analysis, time is the independent variable. When a system is given an excitation, there is a response (output).**

- **Definition:** The response of a system to an applied excitation is called "**Time Response**" and it is a function of c(t).

- Time Response - Example

The response of motor's speed when a command is given to increase the speed is shown in figure,



As seen from figure, the motors speed gradually picks up from 1000 rpm and moves towards1500 rpm. It overshoots and again corrects itself and finally settles down at the last value

Time Response

Generally speaking, the response of any system thus has two parts

(i) Transient Response
(ii) Steady State Response

- That part of the time response that goes to zero as time becomes very large is called as "**Transient Response**"

i.e. $$\lim_{t \to \infty} c(t) = 0$$

- As the name suggests that transient response remains only for some time from initial state to final state.

- **From the transient response we can know;**

✓ When system begins to respond after an input is given.

✓ How much time it takes to reach the output for the first time.

✓ Whether the output shoots beyond the desired value & how much.

✓ Whether the output oscillates about its final value.

✓ When does it settle to the final value.

Steady State Response

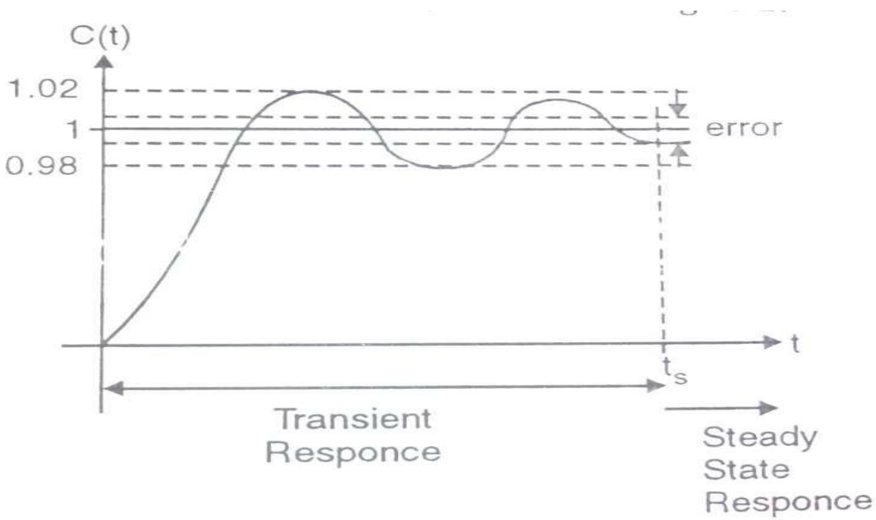
- That part of the response that remains after the transients have died out is called "**Steady State Response**".

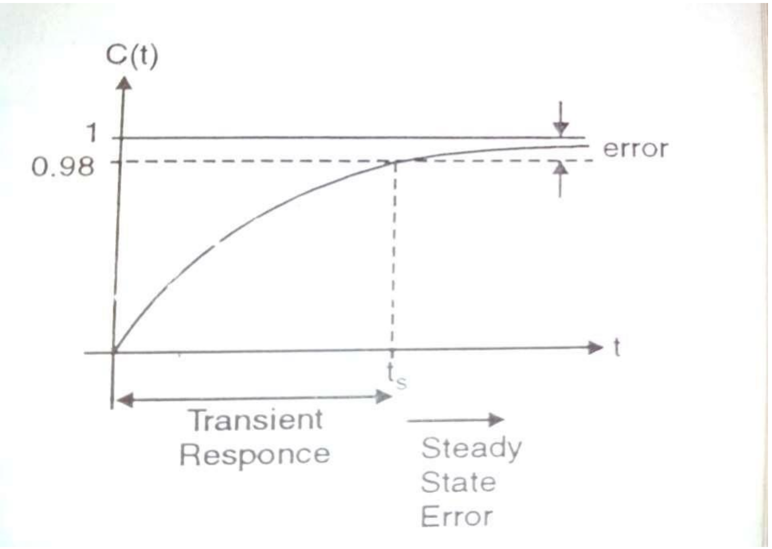
- **From the steady state we can know;**

  ✓ How long it took before steady state was reached.

  ✓ Whether there is any error between the desired and actual values.

  ✓ Whether this error is constant, zero or infinite i.e. unable to track the input.





Standard Test Signal

- It is very interesting fact to know that most control systems do not know what their inputs are going to be.
- Thus system design cannot be done from input point of view as we are unable to know in advance the type input

Need of Standard Test Signal

- From example;
  - ✓ When a radar tracks an enemy plane the nature of the enemy plane's variation is random.
  - ✓ The terrain, curves on road etc. are random for a drives in an automobile system.
  - ✓ The loading on a shearing machine when and which load will be applied or thrown of.

Need of Standard Test Signal

- **Thus from such types of inputs we can expect a system in general to get an input which may be;**
  a) A sudden change
  b) A momentary shock
  c) A constant velocity
  d) A constant acceleration

- **Hence these signals form standard test signals. The response to these signals is analyzed. The above inputs are called as,**
  a) Step input - Signifies a sudden change
  b) Impulse input – Signifies momentary shock
  c) Ramp input – Signifies a constant velocity
  d) Parabolic input – Signifies constant acceleration

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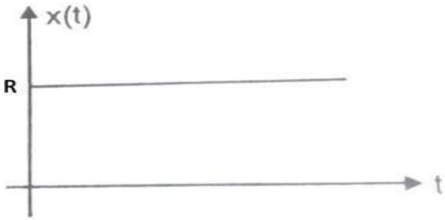
Standard Test Signal

Step Input

Mathematical Representations

$$r(t) = R.\,u(t) \qquad t>0$$
$$= 0 \qquad t<0$$

Graphical Representations



This signal signifies a sudden change in the reference input r(t) at time t=0

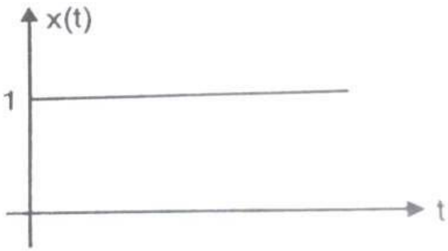
Laplace Representations $\quad L\{R\,u(t)\} = \dfrac{R}{s}$

Unit Step Input

Mathematical Representations

$$r(t) = 1.\,u(t) \qquad t>0$$
$$= 0 \qquad t<0$$

Graphical Representations



This signal signifies a sudden change in the reference input r(t) at time t=0

Laplace Representations $\quad L\{u(t)\} = \dfrac{1}{s}$

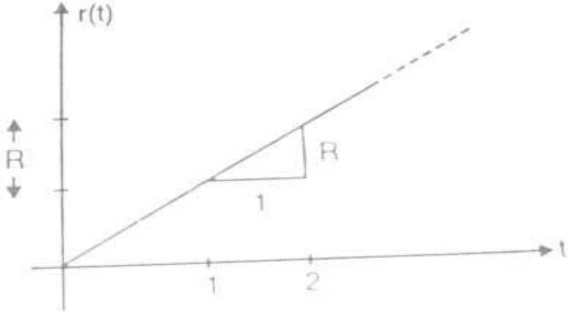
Standard Test Signal

Ramp Input

Mathematical Representations

$$r(t) = R.t \qquad t>0$$
$$= 0 \qquad t<0$$

Graphical Representations



Signal have constant velocity i.e. constant change in it's value w.r.t. time

Laplace Representations

$$L\{Rt\} = \frac{R}{s^2}$$

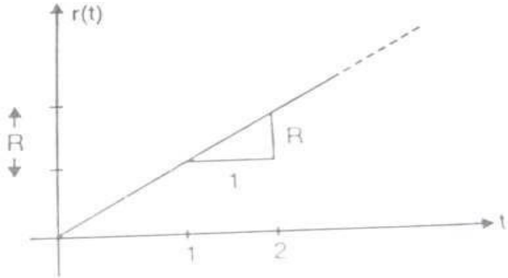
Unit Ramp Input

Mathematical Representations

$$r(t) = 1.t \qquad t>0$$
$$= 0 \qquad t<0$$

If R=1 it is called a unit ramp input

Graphical Representations



Laplace Representations

$$L\{Rt\} = \frac{1}{s^2}$$

Standard Test Signal

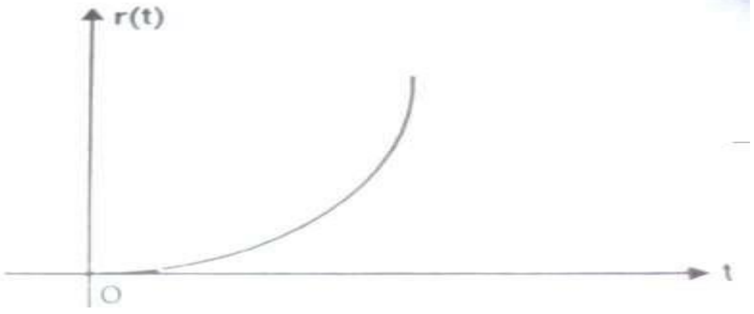
Parabolic Input

Mathematical Representations

$$r(t) = \frac{Rt^2}{2} \qquad t>0$$

$$= 0 \qquad t<0$$

Graphical Representations



Laplace Representations $L\{Rt\} = \frac{R}{s^3}$

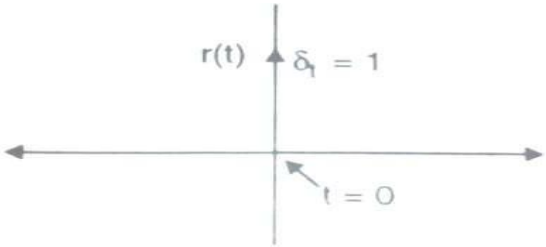
Impulse Input

Mathematical Representations

$$r(t) = \delta(t) = 1 \qquad t>0$$

$$= 0 \qquad t<0$$

Graphical Representations



The function has a unit value only for t=0. In practical cases, a pulse whose time approaches zero is taken as an impulse function.

Laplace Representations $L\{\delta(t)\} = 1$

Poles & Zeros of Transfer Function

The transfer function is given by,

$$G(s) = \frac{C(s)}{R(s)}$$

Both C(s) and R(s) are polynomials in s

$$\therefore G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots\ldots\ldots\ldots + b_o}{s^n + a_{n-1} s^{n-1} + \ldots\ldots\ldots\ldots\ldots\ldots + a_n}$$

$$= \frac{K(s - b_1)(s - b_2)(s - b_3)\ldots\ldots\ldots\ldots(s - b_m)}{(s - a_1)(s - a_2)(s - a_3)\ldots\ldots\ldots\ldots(s - a_n)}$$

Where, K= system gain
n= Type of system

- **Poles:** The values of 's', for which the transfer function magnitude |G(s)| becomes infinite after substitution in the denominator of the system are called as **"Poles"** of transfer function.
- **Zeros:** The values of 's', for which the transfer function magnitude |G(s)| becomes zero after substitution in the numerator of the system are called as "**Zeros**" of transfer function.

Pole- Zero Plot

- The diagram obtained by locating all poles and zeros of the transfer function in the s-plane is called as “Pole-zero plot”.
- The s-plane has two axis real and imaginary. Since $s = \sigma + j\omega$, the X-axis stands for real axis and shows a value of σ
- Similarly, Y-axis stands for $j\omega$ and represents the imaginary axis.

Characteristics Equation

Definition: The equation obtained by equating the denominator polynomial of a transfer function to zero is called as the “**Characteristics Equation**”

$$s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots\ldots\ldots\ldots\ldots + a_n$$

Example 1

For the given transfer function,

$$T.F. = \frac{K(s+6)}{s(s+2)(s+5)(s^2+7s+12)}$$

Find: (i) Poles (ii)Zeros
(iii) Pole-zero Plot (iv) Characteristics Equation

Solution: (i)Poles

The poles can be obtained by equating denominator with zero

$$s(s+2)(s+5)(s^2+7s+12) = 0$$

$$\therefore s = 0$$

$$\therefore s+2 = 0 \qquad \therefore s = -2$$

$$\therefore s+5 = 0 \qquad \therefore s = -5$$

Example 1 Cont

$$s(s+2)(s+5)(s^2+7s+12)=0$$

$$(s^2+7s+12)=(s+3)(s+4)$$

$$\therefore s+3=0 \qquad \therefore s=-3$$

$$\therefore s+4=0 \qquad \therefore s=-4$$

The poles are s=0, -2, -3, -4, -5

(ii) Zeros:

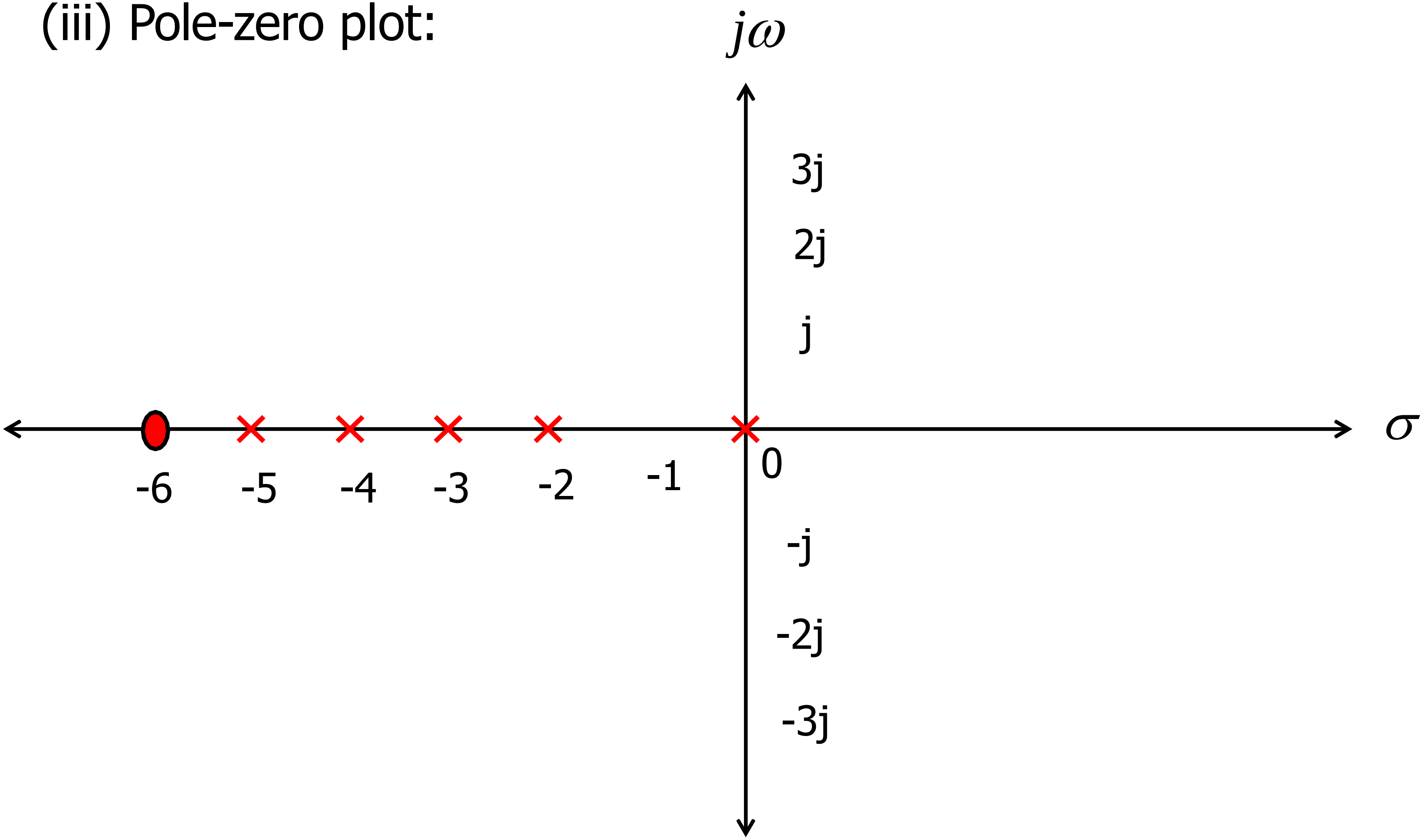
The zeros can be obtained by equating numerator with zero

$$s+6=0 \qquad \therefore s=-6$$

The zeros are s=-6

Example 1 Cont

(iii) Pole-zero plot:



(iv) Characteristics Equation:

$$s(s+2)(s+5)(s^2+7s+12)=0$$

$$s(s^2+7s+10)(s^2+7s+12)=0$$

$$\therefore (s^3+7s^2+10s)(s^2+7s+12)=0$$

$$\therefore s^5+7s^4+12s^3+7s^4+49s^3+84s^2+10s^3+70s^2+120s=0$$

$$\therefore s^5+14s^4+71s^3+154s^2+120s=0$$