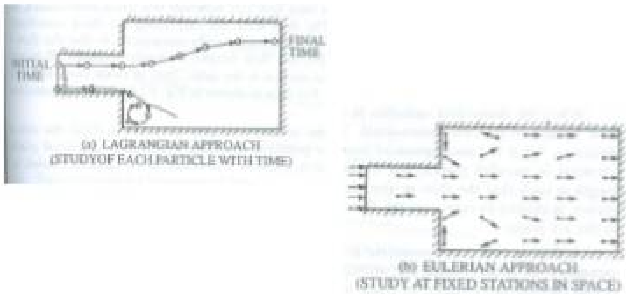
Unit 2

Fluid Kinematics

Introduction

Fluid Description

- **Kinematics:** The study of motion.
- **Fluid kinematics:** The study of how fluids flow and how to describe fluid motion.
- There are distinct ways to describe motion of fluid particles:
  a) Lagrangian
  b) Eulerian



(a) LAGRANGIAN APPROACH (STUDY OF EACH PARTICLE WITH TIME)

(b) EULERIAN APPROACH (STUDY AT FIXED STATIONS IN SPACE)

Euler and Lagrange descriptions

- Euler approach The fluid properties p, ρ, v are written as functions of space and times. The flow is determined by the analyzing the behavior of the functions.

- Lagrange approach Pieces of the fluid are “tagged”. The fluid flow properties are determined by tracking the motion and properties of the particles as they move in time.

Types of Fluid Flow

Variation Of Flow Parameters

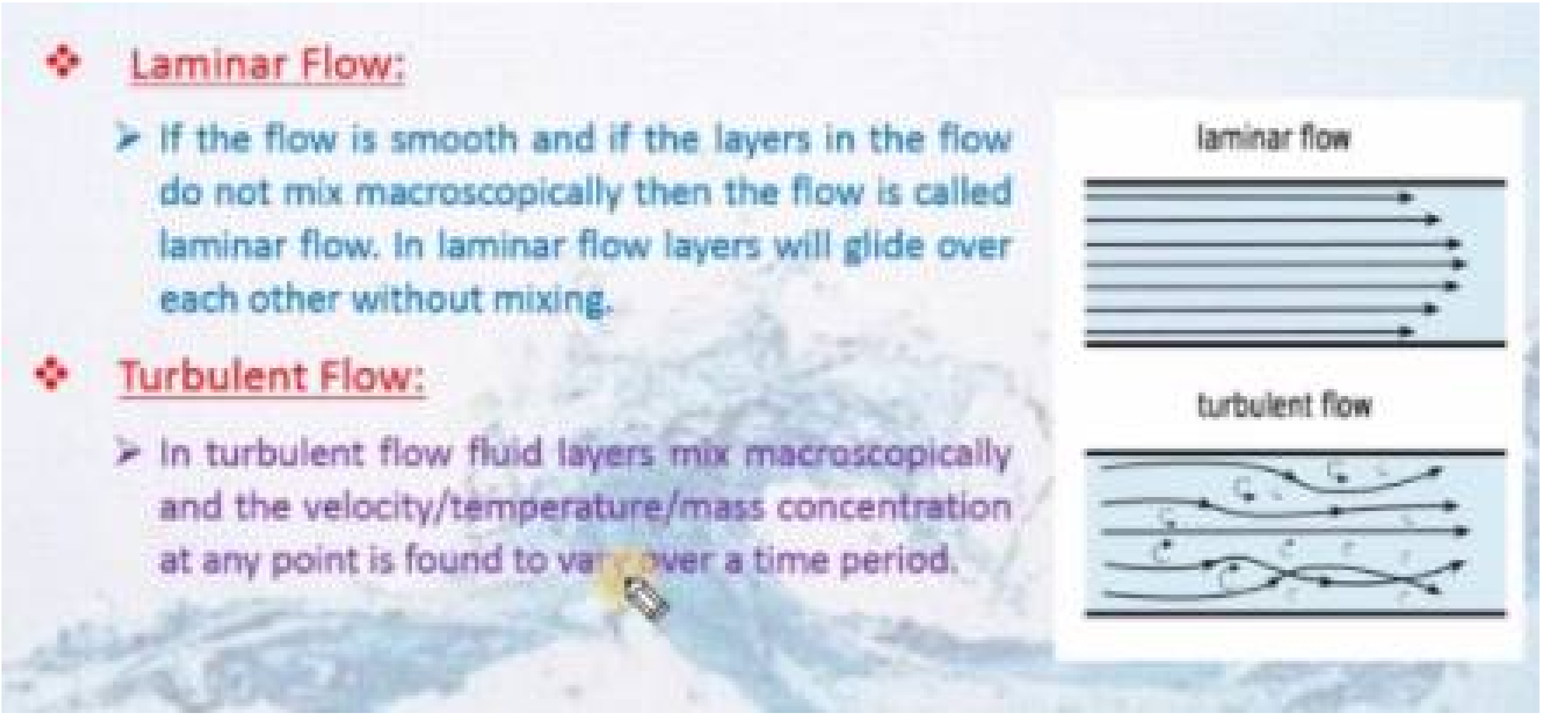
Steady and Unsteady flow

- Steady flow is defined as the flow in which pressure and density do not change with time in a control volume
- In the Lagrangian approach, time is inherent in describing the trajectory of any particle. But in steady flow, the velocities of all particles passing through any fixed point at different times will be same.

Uniform and Non-uniform flow

- When velocity and other hydrodynamic parameters at any instant of time do not change from point to point in a flow field, the flow is said to be uniform. Hence for a uniform flow, the velocity is a function of time only.

Laminar And Turbulent Flow



Compressible and Incompressible Flow

- The flow in which the density does not remain constant for the fluid flow is called as compressible flow.

- E.g. problems involving flight of rockets, aircrafts, flow fo air in problems concerned with tubomachines, compressor blades, flow of gases through openings like nozzles.

- The flow in which the density is constant for the fluid flow is called as incompressible flow.

- E.g. problems involving liquids i.e. hydraulics problems, flow of gases in machines like fans and blowers.

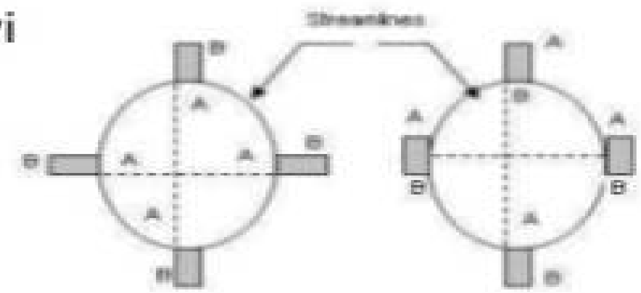
Rotational and Irrigational Flow

- The flow in which the fluid particle while flowing along stream lines, also rotate about their own axis is called as rotational flow.

- E.g. motion of liquid in a rotating cylinder (forced vortex) as rotational flow.

- The flow in which the fluid particle while flowing along streamlines, do not rotate about their own axis is called as irrigational flow.

- E.g. flow of liquid in an emptyi vortex) as a rotational flow.



One-, Two- and Three-Dimensional Flow

- The flow in which the velocity is the function of time and one space co-ordinate (x) is called as One-dimensional flow.

- E.g. flow through the pipe is consider as a one dimensional flow.

$u = f(x), \qquad v = 0, \qquad w = 0$

- The flow in which the velocity is the function of time and to space co-ordinate (x,y) is called as two-dimensional flow.

- Eg viscous flow between parallel plates of large extent, flow at the middle part of airplane wing, flow over a long spillway, flow below long weirs are consider as two-dimensional flow.

$u = f_1(x,y)$, $v = f_2(x,y)$, $w = 0$

- The flow is converging or diverging pipes or open channels are as three dimensional flow. Flow in a river, flow at a inlet of a nozzle etc. are the example of three-dimensional flow.

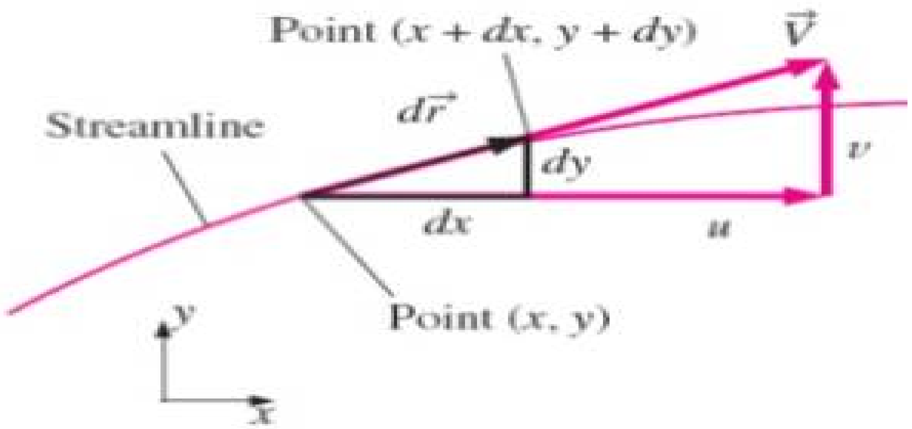
$u = f_1(x,y,z)$, $v = f_2(x,y,z)$, $w = f_3(x,y,z)$

Streamline and Stream-tube

Streamline: A curve that is everywhere tangent to the instantaneous local velocity vector.

Stream line at any instant can be defined as an imaginary curve or line in the flow field, so that the tangent to the curve at any point, represents the direction of the instantaneous velocity at that point.

Streamlines are useful as indicators of the instantaneous direction of fluid motion throughout the flow field.



For two-dimensional flow in the *xy*-plane, arc length $d\vec{r} = (dx, dy)$ along a *streamline* is everywhere tangent to the local instantaneous velocity vector $\vec{V} = (u, v)$.

Properties of Streamlines

- The component of velocity, normal to a streamline is zero, there can be no flow across a streamline.
- Since the instantaneous velocity at a point in a fluid flow must be unique in magnitude and direction, the same point cannot belong to more than one streamline.
- *In other words, a streamline cannot intersect itself nor can any streamline intersect another streamline.*
- In a steady flow, the orientation or the pattern of streamlines will be fixed.
- In an unsteady flow where the velocity vector changes with time, the pattern of streamlines also changes from instant to instant.

Equation of Streamlines

- Consider a streamline in a plane flow in the x-y plane. By definition, the velocity vector U at a point P must be tangential to the streamline at that point. It follows that

$$\frac{dy}{dx} = tan\ \theta = \frac{v}{u} \quad ; \quad u\ dy - v\ dx = 0$$

where u and v are velocity components along x and y directions respectively. The velocity vector is expressed as

$$U = U\ (s, t)$$

This shows that the velocity may vary along a streamline direction as well as with the passage of time.

- Consider an elementary displacement element along a general streamline where the velocity U such that

$$U = ui + vj + wk \qquad \delta \mathbf{s} = (\delta \mathbf{x})\, i + (\delta \mathbf{y})\, j + (\delta z)\, k$$

$$U \ \mathbf{x}\ \delta \mathbf{s} = \mathbf{0} \qquad \text{or} \qquad \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ \mathrm{u} & \mathrm{v} & \mathrm{w} \\ d\mathrm{x} & d\mathrm{y} & d\mathrm{z} \end{vmatrix} = 0$$

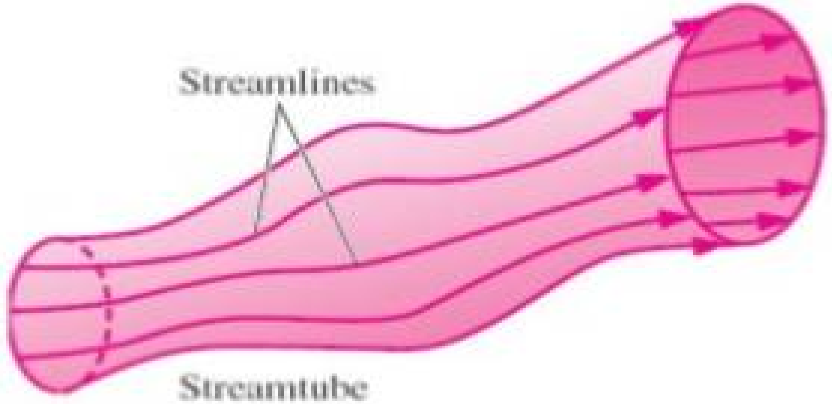
i.e., $(\mathbf{v}\delta \mathbf{z} - \mathbf{w}\delta \mathbf{y})\, i - (u\delta z - w\delta x)\, j + (u\delta y - v\delta x)\, k = 0$

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

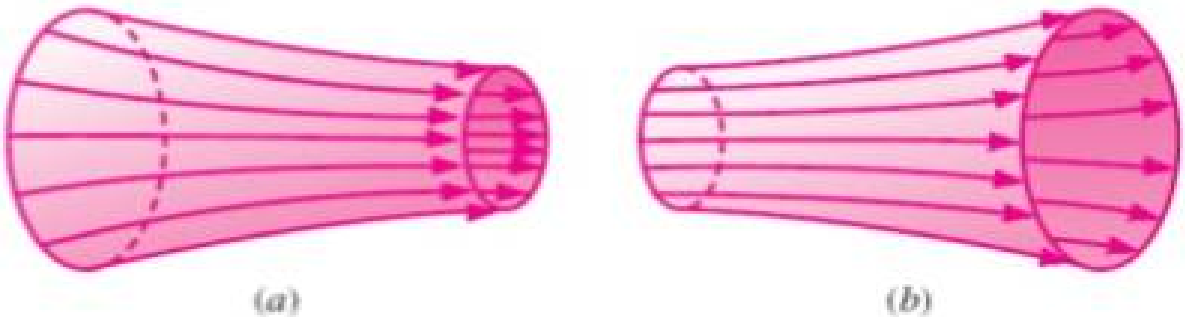
which is the **equation of a streamline.**

- Stream tube: A bundle of neighboring stream lines may be imagined to form a passage through which the fluid flows. This passage (non necessarily circular in cross-section) is known as a stream tube.
- Since a stream tube is bounded on all sides by streamlines, velocity does not exist across a streamline, no fluid may enter or leave a stream tube except through its ends.

A stream tube consists of a bundle of streamlines much like a communications cable consists of a bundle of fiber-optic cables.

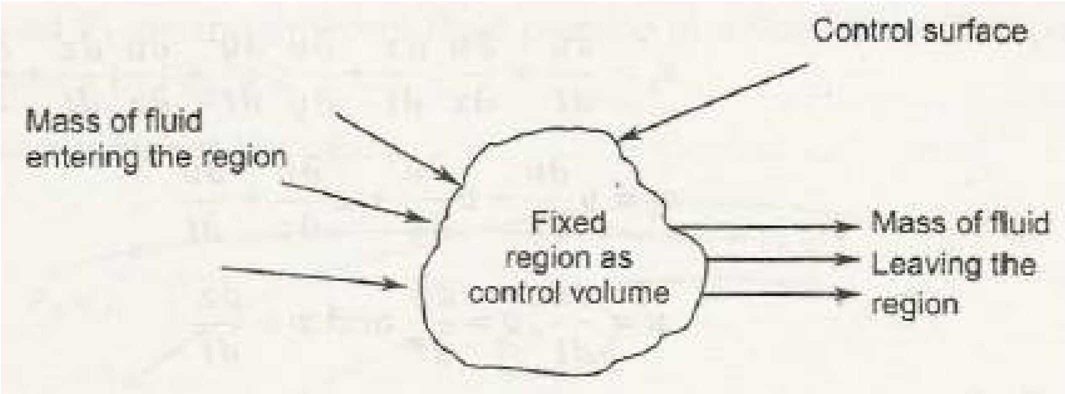


Both streamlines and stream tubes are instantaneous quantities, defined at a particular instant in time according to the velocity field at that instant.



In an incompressible flow field, a stream tube (*a*) decreases in diameter as the flow accelerates or converges and (*b*) increases in diameter as the flow decelerates or diverges.

Continuity Equation in Three Dimensions in a Differential Form



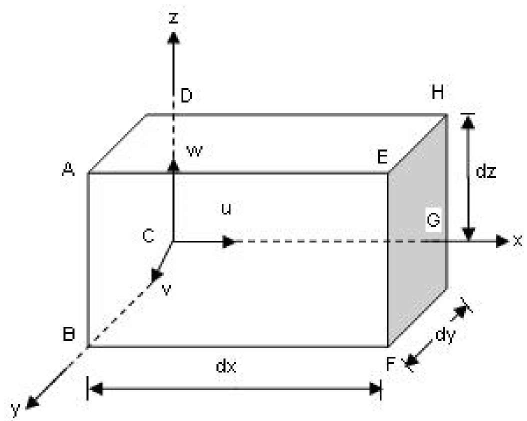
Above fig explains the concept of control volume and control surface.

When fluid flow through a full pipe, the volume of fluid entering into the pipe must be equal to the volume of the fluid leaving the pipe, even if the diameter of the pipe vary.

Therefore we can define the continuity equation as the equation based on the principle of conservation of mass.

Therefore, for a flowing fluid through the pipe at every cross-section, the quantity of fluid per second will be constant.

Let us consider we have one pipe through which fluid is flowing. Let us consider a fluid element of having length dx, dy and dz in the direction of X, Y and Z respectively.



Mass of fluid entering the face ABCD per second = Density x velocity in x direction x Area ABCD

Mass of fluid entering the face ABCD per second = ρ x u x dy.dz = ρ u dy.dz

Mass of fluid leaving the face EFGH per second = (ρ u dy.dz) + (∂ / ∂x) (ρ u dy.dz) dx

Gain of mass = Mass flow rate in to the system – Mass for rate out of the system

Gain of mass = Mass of fluid entering the face ABCD per second - Mass of fluid leaving the face EFGH per second

Therefore, we will have following equation for mass gain in X-direction

Gain of mass in X-direction = ρ u dy.dz – [(ρ u dy.dz) + (∂ / ∂x) (ρ u dy.dz) dx]

Gain of mass in X-direction = - (∂ / ∂x) (ρ u dx.dy.dz)

Similarly, we will have following equations for mass gain in Y-direction and Z-direction

Gain of mass in Y-direction = $- (\partial / \partial y) (\rho \, v \, dx.dy.dz)$

Gain of mass in Z-direction = $- (\partial / \partial z) (\rho \, w \, dx.dy.dz)$

Net gain of masses = $- [\partial / \partial x (\rho \, u) + \partial / \partial y (\rho \, v) + \partial / \partial z (\rho \, w)] \, dx.dy.dz$

As per the principle of conservation of mass, mass could not be created or destroyed in the fluid element. Therefore, net increase of mass per unit time in the fluid element should be equal to the rate of increase of mass in the fluid element.

Mass of fluid in the fluid element = $\rho \, dx.dy.dz$

Rate of increase of mass in the fluid element = $\partial \rho / \partial t. \, dx.dy.dz$

Therefore,

$$- [\partial / \partial x (\rho u) + \partial / \partial y (\rho v) + \partial / \partial z (\rho w)] dx.dy.dz = \partial\rho/\partial t. dx.dy.dz$$

$$- [\partial / \partial x (\rho u) + \partial / \partial y (\rho v) + \partial / \partial z (\rho w)] = \partial\rho/\partial t$$

$$\partial\rho/\partial t + \partial / \partial x (\rho u) + \partial / \partial y (\rho v) + \partial / \partial z (\rho w) = 0$$

Above equation is the continuity equation in Cartesian co-ordinates in its most general form. This equation will be applicable to following types of fluid flow.

1. Steady and un-steady flow
2. Uniform and non-uniform flow
3. Compressible and incompressible flow

For steady flow, continuity equation will be as mentioned here

For steady flow, $\partial\rho/\partial t = 0$

$$\partial / \partial x (\rho u) + \partial / \partial y (\rho v) + \partial / \partial z (\rho w) = 0$$

For incompressible flow, continuity equation will be as mentioned here

For incompressible flow, ρ will be constant and we will have following continuity equation

$$\partial u / \partial x + \partial v / \partial y + \partial w / \partial z = 0$$

Above equation is the continuity equation in three dimensions

Stream Function

It is defined as the scalar function of space and time such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. It is denoted by ψ *(psi)* and defined only for two dimensional flow. Mathematically for steady flow it is defined as $\psi = f(x, y)$ such that

$$u = \frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial \psi}{\partial x}$$

Properties:

a) If the stream function (ψ) exists, it is a possible case of fluid flow which may be rotational or irrotational.

b) If stream function (ψ) satisfies the Laplace equation it is possible case of an irrotational flow.

Velocity potential

Velocity potential function is a scalar function of space and time. If 'phi' is the representation of velocity potential function, then the velocity function for a steady fluid flow is given by the expression,

$$\Phi = f(x, y, z)$$

It is a scalar function, whose negative derivative, with respect to any direction, gives the velocity component in that direction.

$$u = -\frac{\partial \varphi}{\partial x};\ v = -\frac{\partial \varphi}{\partial y};\ w = -\frac{\partial \varphi}{\partial z};$$

Here, **u**, **v**, and **w** are the velocity components of the fluid flow along x, y, and z directions.

Relationship Between Velocity Potential Function and Stream Function

From the expressions of velocity potential function and stream function,

$$u = -\frac{\partial\varphi}{\partial x};\ v = -\frac{\partial\varphi}{\partial y}$$

and

$$\frac{\delta\psi}{\delta x} = v;\ \frac{\delta\psi}{\delta y} = -u;$$

By comparing both the equations, we get

$$\frac{\partial\varphi}{\partial x} = \frac{\delta\psi}{\delta y};$$

$$\frac{\partial\varphi}{\partial y} = -\frac{\delta\psi}{\delta x};$$

Flow Net: Graphical Construction used to calculate groundwater flow through soil. Comprised of Flow Lines and Equipotential Lines.

Flow Line: A line along which a water particle moves through a permeable soil medium.

Flow Channel: Strip between any two adjacent Flow Lines.

Equipotential Lines: A line along which the potential head at all points is equal.