MPM: 203 NUCLEAR AND PARTICLE PHYSICS

UNIT –I: Nuclei And Its Properties

Lecture-6

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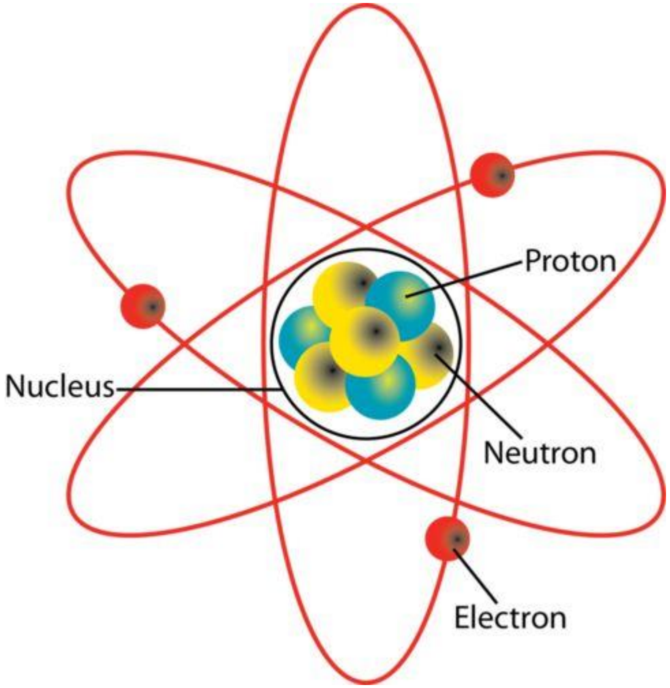
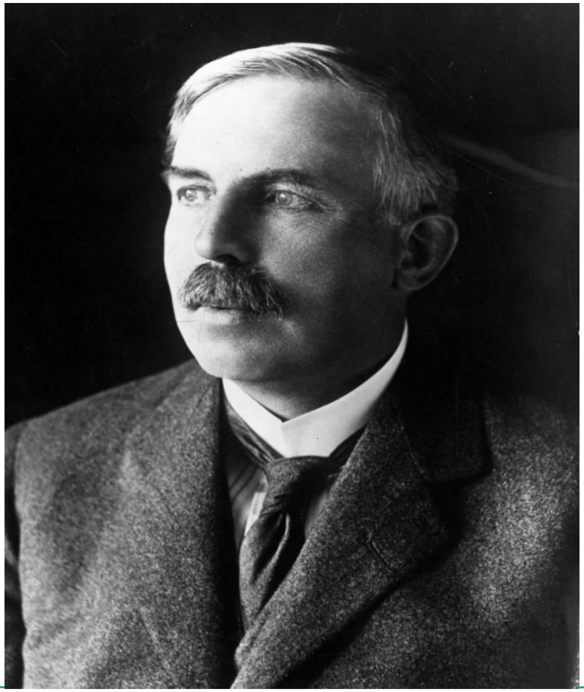




Nuclear Electric Quadrupole Moment

- The quadrupole moment of charge distribution is given by
- $$Q = \frac{e}{2}(3\,z^2 - r^2) \quad \text{------}(1)$$
- This equation has been derived by classical considerations.
- When quantum Mechanics is applied the quadrupole moment receives a new definition which differs from classical definition in following aspect:

1. The quadrupole moment is not taken about the body axis of I^* but about the axis of its maximum projected component m_I = I.
2. The numerical expression ½ in the expression (1) disappears
3. The probability density of proton at any position (x,y,z) with in the nucleus is represented by $|\psi|^2$where ψ is the wavefunction at (x,y,z). The quantum mechanical charge distribution is therefore continuous and can be represented by an average charge density $\rho(x, y, z)$.

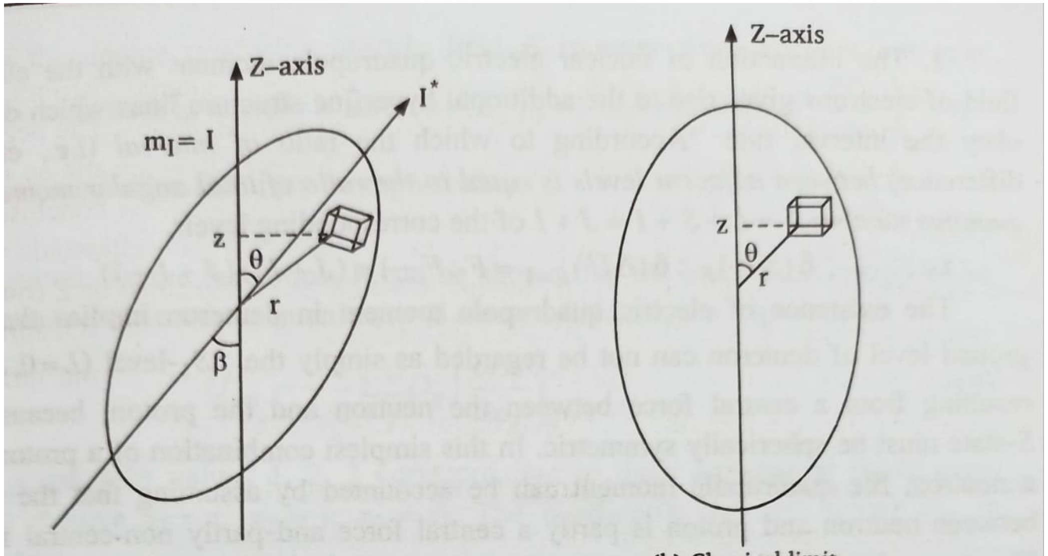
Nuclear Electric Quadrupole Moment

- The Integral over the charge distribution is divided by the proton charge e which makes all nuclear quadrupole moments to possess the dimensions of m^2 $only$.
- Thus if ρ is the nuclear charge density in the volume element $\mathrm{d}\tau$ at point (z, r) the nuclear quadrupole moment will be defined as the average of Q = $\frac{1}{e}\int\rho$ (3 z^2- r^2) $\mathrm{d}\tau$, taken about m_I = I.

- Since I^* is the precesses about I
- From the fig z= r cosθ, thus, Q = $\frac{1}{e}\int \rho r^2$ (3 cos^2 θ - 1) $\mathrm{d}\tau$
- This may be written as Q = $\frac{1}{e}[\rho r^2$(3 cos^2 $\theta - 1)]_{av}$ ----------(3)
- If β is the angle made by body axis with Z-axis in space, then

Nuclear Electric Quadrupole Moment

- Cosβ = $\frac{m_I}{I^*}$ = $\frac{m_I}{\sqrt{I(I+1)}}$ -----(4)
- Where m_I is magnetic quantum number
- It can be shown that the effective value of the quadrupole moment is proportional to (3 $cos^2\beta$− 1)



(a) Quantum mechanical (b) Classical limit

Then the effective value of quadrupole moment Q (m_I) in the state m_I is related to the value of Q in the state m_I= I by

$$Q\ (m_I) = \frac{3\ cos^2\beta_m - 1}{3\ cos^2\beta_I - 1} Q = \frac{3m_I^2 - I(I+1)}{I(2I-1)} Q \text{---------(5)}$$

Nuclear Electric Quadrupole Moment

- From equation (5) it can be seen that the nuclei having I=0 or I=1/2 can exhibit no quadrupole moment

- More Physically by noting that for I=1/2, Cos$\beta_l = \frac{\frac{1}{2}}{\sqrt{\frac{1}{2}\times\frac{3}{2}}} = \frac{1}{\sqrt{3}}$ and from symmetry consideration, the average value of $(3\,cos^2\boldsymbol{\theta} - 1)$ in equation (3) becomes zero.

- This does not mean that nuclei with I= ½ have perfectly spherical distributions of charge about their body axis I^* but only that the maximum observable component of Q is zero.

- Finite quadrupole moment must exist only for nuclei having I≥ 1.

PARITY

- Parity is a property of wave function describing the quantum mechanical system.
- In Quantum mechanics, the physical description of a nuclear particle is described by a wave function **where** $\psi(x, y, z)$ which depends on position (x,y,z) and the spin (s) of the particle.
- The probability of finding the particle at position (x,y,z) and with a spin – orientation(s) is given by $\psi^* \psi$ or $|\psi|^2$.
- Where $|\psi|$ is the modulus of ψ and ψ^*, the complex conjugate of ψ.
- More correctly ψ (x,y,z,s) may be taken as product of two wave functions ψ_1(x, y, z) and ψ_2(s), where ψ (x,y,z) depends only on space coordinates (x, y, z) and ψ_2(s) depends only on spin orientation.

PARITY

- If the sign of the wave function $\psi_1(x, y, z)$ does not change by reflection of particle through the origin, the parity of the particle is said to be even, but if the wave-function $\psi_1(x, y, z)$ changes sign by reflection through the origin, the parity is said to be odd.
- Thus
- $\psi_1(x, y, z) = \psi_1(-x, -y, -z)$ represents even parity
- $\psi_1(x, y, z) = -\psi_1(-x, -y, -z)$ represents odd parity

- Equivalently
- $\psi_1(x, y, z, s) = \psi_1(-x, -y, -z, s)$ represents even parity
- $\psi_1(x, y, z, s) = -\psi_1(-x, -y, -z, s)$ represents odd parity

PARITY

- A wave-function describing a number of particles can be written as the product of individual particles wave functions $\psi = \psi_1\ \psi_2\ \psi_3\ \psi_4$ ……or the linear combination of such product.

- Obviously, the parity of whole system is the product of parities of single particle wave function

- Since $|\psi|^2$ is symmetric whether the parity is even or odd, the mass density and the charge density of the nuclei are always symmetric.

- It can be shown for no-relativistic system that the spatial part of wave-function ψ, on reflection of particle about the origin does not change sign if the angular momentum quantum number l is even but it changes sign if l is odd.

PARITY

- *Hence for a particle with an even value of angular momentum quantum number l, the parity is even and with an odd value of l the parity is odd.*
- For a system of particles if the algebraic sum of the individual numerical values of l for all particles (i.e. $\sum l_i$) is even, then parity is even but if $\sum l_i$ is odd, the parity is odd.
- Accordingly a system containing an even number of odd parity particles and any number of even parity particles will have even parity; while a system containing an odd number of odd parity particles and any number of even parity particles will have an odd parity.
- To represent even parity the superscript (+) is used on total nuclear quantum number I, while superscript (-) on I represents odd parity.

PARITY

- For example, I = 1^{+} denotes even parity for ${}_{1}H^{2}$ while I= 7^{-} denotes odd parity (for ${}_{71}Lu^{176}$).

- In Quantum Mechanics the parity operator means a reflection operator about the plan of the symmetry, the eigen value equation is
- $P\psi = \lambda\psi$

- Where P is an operator acting on state ψ and λ is the eigen value.
- If P represents parity operator, then λ = $\pm$ 1, so that
- $P\psi = (\pm 1)\psi$

- Thus, there are two eigen values of parity operator +1 and -1.
- The +1 eigen value correspond to even parity and -1 eigen value to odd parity.

PARITY

- Parity is purely quantum mechanical concept and has no simple analogy in classical physics.

- The intrinsic parity of electron is arbitrary taken as even (or positive).

- From the experimental study of simple system, the parity of the proton, neutron and neutrino is same as that of electron, hence it is even,

- On the other hand π – meson is found to have odd intrinsic parity.

CHANGE OF PARITY

- In nuclear processes the parity is normally conserved like total energy, linear momentum and angular momentum.
- In 1957 Lee and Yang has suggested that parity is not conserved in weak- interactions like beta-decay and muon decay.
- Thus parity may change in certain nuclear processes.
- The parity of nucleus changes whenever there is emission or absorption of photons or particles of odd total parity.
- Conversely if in any nuclear process the parity of system changes, it means that it has emitted or absorbed the photons or particles of odd total parity,

Mass Defect and Packing Fraction

- The **atomic number,** *Z* (sometimes called the *charge number*), which equals the number of protons in the nucleus
- The **neutron number,** *N*, which equals the number of neutrons in the nucleus.
- The **mass number,** *A*, which equals the number of nucleons (neutrons plus protons) in the nucleus.

The isotopes of an element have the same *Z* value but different *N* and *A* values.

The natural abundances of isotopes can differ substantially. For example, ${}^{11}_{6}C$, ${}^{12}_{6}C$, ${}^{13}_{6}C$, and ${}^{14}_{6}C$ are four isotopes of carbon. The natural abundance of the ${}^{12}_{6}C$ isotope is about 98.9%, whereas that of the ${}^{13}_{6}C$ isotope is only about 1.1%. Some isotopes do not occur naturally but can be produced in the laboratory through nuclear reactions. Even the simplest element, hydrogen, has isotopes: ${}^{1}_{1}H$, the ordinary hydrogen nucleus; ${}^{2}_{1}H$, deuterium; and ${}^{3}_{1}H$, tritium.

Important Parameters

Table 13.1 Masses of the Proton, Neutron, and Electron in Various Units

	Mass		
Particle	**kg**	**u**	**MeV/c^2**
Proton	$1.672\,623 \times 10^{-27}$	1.007 276	938.272 3
Neutron	$1.674\,929 \times 10^{-27}$	1.008 665	939.565 6
Electron	$9.109\,390 \times 10^{-31}$	$5.48\,579\,9 \times 10^{-4}$	0.510 999 1

$$r = r_0 A^{1/3}$$

Table 13.2 Masses, Spins, and Magnetic Moments of the Proton, Neutron, and Electron

Particle	Mass (MeV/c^2)	Spin	Magnetic Moment
Proton	938.28	$\frac{1}{2}$	$2.7928\mu_n$
Neutron	939.57	$\frac{1}{2}$	$-1.9135\mu_n$
Electron	0.510 99	$\frac{1}{2}$	$-1.0012\mu_B$

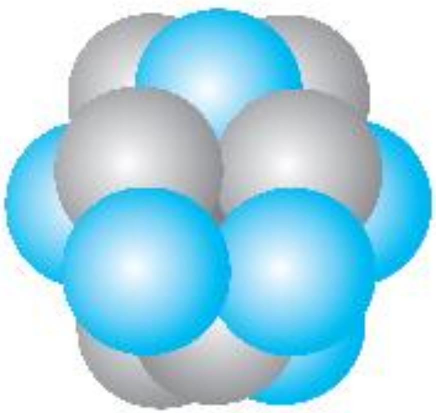


Figure 13.3 A nucleus can be modeled as a cluster of tightly packed spheres, each of which is a nucleon.

The nuclear density is approximately 2.3×10^{14} times as great as the density of water ($\rho_{\text{water}} = 1.0 \times 10^3$ kg/m^3)!