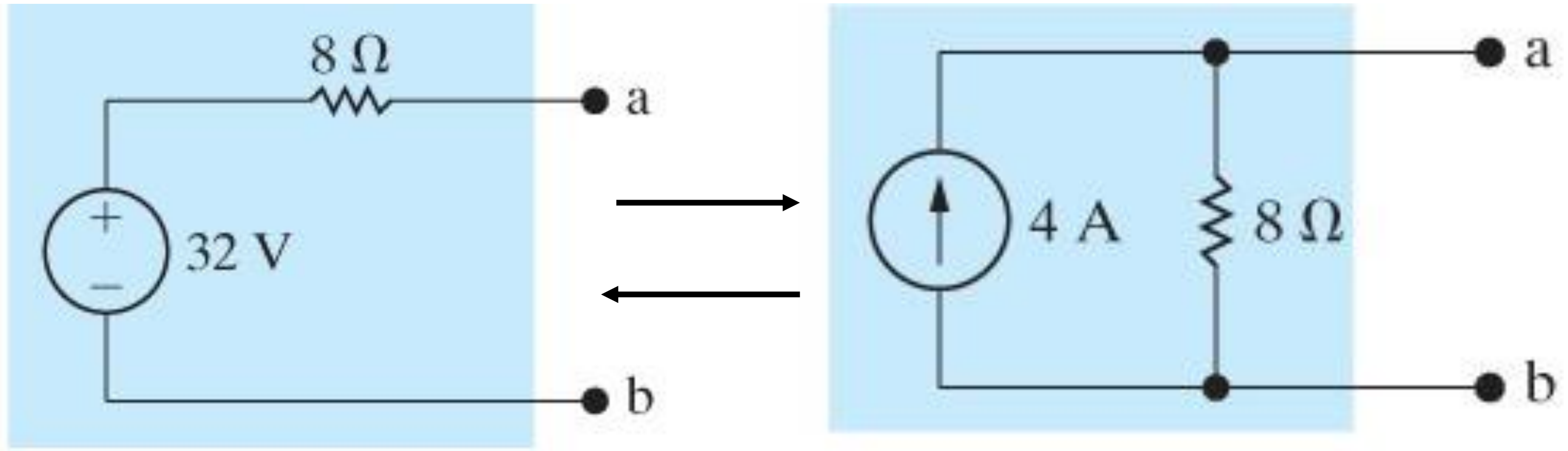
The Norton Equivalent Circuit

- Get the Norton Equivalent Circuit from the Thevenin by Source Transformation.



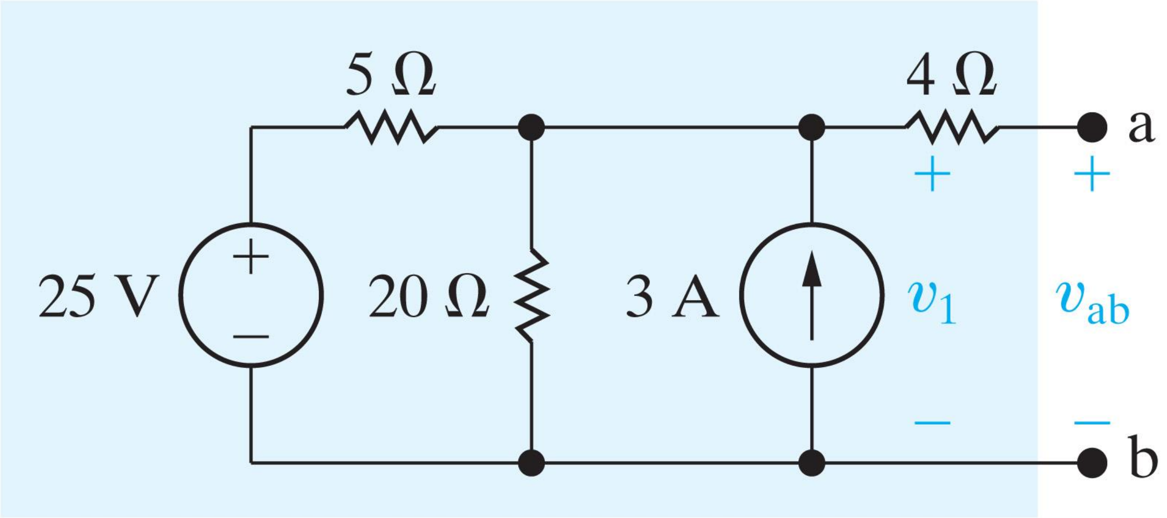
Alternate Way to Determine the Thevenin Resistance

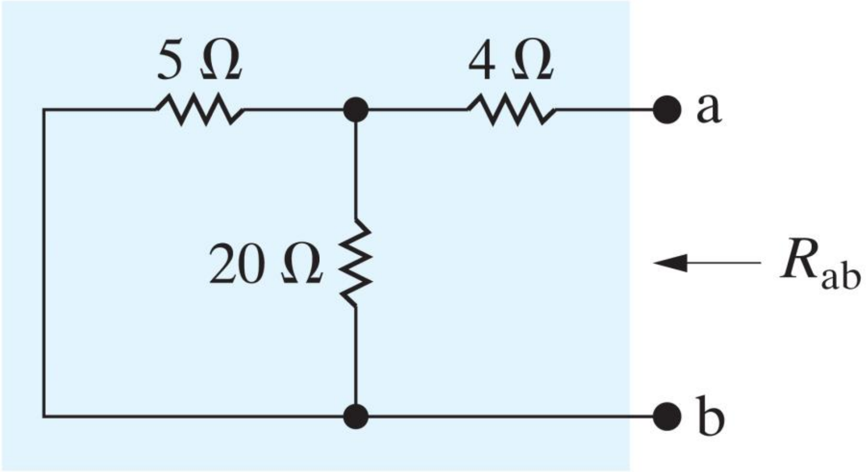
If the sources are all Independent

If the Sources Are All Independent

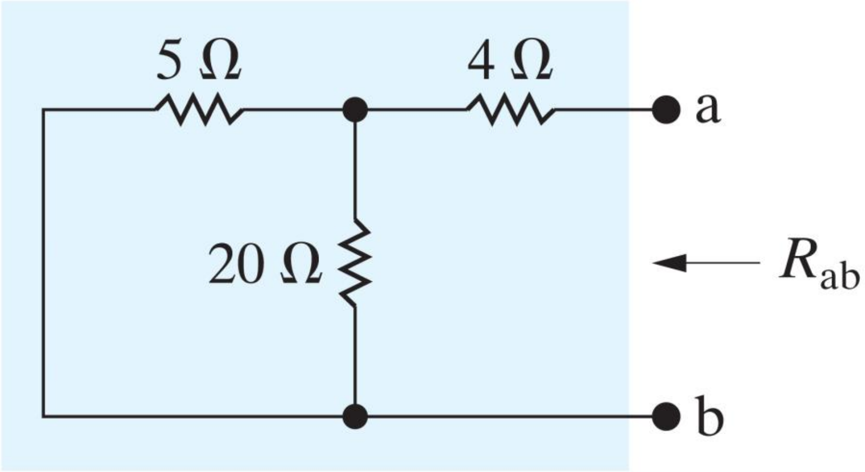
- Look into the a-b terminals with all sources set equal to 0.
  - Voltage Sources go to Short Circuits
  - Current Sources go to Open Circuits
- Determine the resistance

For our Example





Looking into the a-b terminals



$$R_{ab} = R_{Th} = 4\Omega + (5\Omega \, P \, 20\Omega)$$

$$R_{ab} = R_{Th} = 4 + \frac{(5)(20)}{5+20} = 8\Omega$$

APPLICATION OF THE LAPLACE TRANSFORM TO CIRCUIT ANALYSIS

LEARNING GOALS

Laplace circuit solutions
Showing the usefulness of the Laplace transform

Circuit Element Models
Transforming circuits into the Laplace domain

Analysis Techniques
All standard analysis techniques, KVL, KCL, node, loop analysis, Thevenin's theorem are applicable

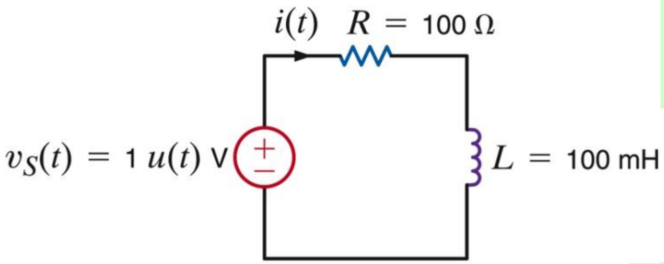
Transfer Function
The concept is revisited and given a formal meaning

Pole-Zero Plots/Bode Plots
Establishing the connection between them

Steady State Response
AC analysis revisited

LAPLACE CIRCUIT SOLUTIONS

We compare a conventional approach to solve differential equations with a technique using the Laplace transform



KVL: $v_S(t) = Ri(t) + L\dfrac{di}{dt}(t)$

Complementary

$i = i_C + i_p$

Particular

Complementary equation

$$Ri_C(t) + L\frac{di_C}{dt}(t) = 0 \Rightarrow i_C(t) = K_C e^{-\alpha t}$$

$$RK_C e^{-\alpha t} + LK_C(-\alpha e^{-\alpha t}) = 0 \Rightarrow \alpha = \frac{R}{L}$$

Particular solution for this case

$$i_p(t) = K_p \qquad \Rightarrow v_S = 1 = RK_p$$

$$i(t) = \frac{1}{R} + K_C e^{-\frac{R}{L}t}$$

Use boundary conditions

$$v_S(t) = 0 \; for \; t < 0 \Rightarrow \; i(0) = 0$$

$$i(t) = \frac{1}{R}\left(1 - e^{-\frac{R}{L}t}\right); \; t > 0$$

"Take Laplace transform" of the equation

$$v_S(t) = Ri(t) + L\frac{di}{dt}(t)$$

$\mathfrak{L}$

$$V_S(s) = RI(s) + L\mathcal{L}\left[\frac{di}{dt}\right]$$

$$\mathcal{L}\left[\frac{di}{dt}\right] = sI(s) - i(0) = sI(s)$$

Initial conditions are automatically included

$$\therefore \frac{1}{s} = RI(s) + LsI(s) \qquad I(s) = \frac{1}{s(R + Ls)}$$

$$I(s) = \frac{1/L}{s(R/L + s)} = \frac{K_1}{s} + \frac{K_2}{s + R/L}$$

Only algebra is needed

$$K_1 = sI(s)|_{s=0} = \frac{1}{R}$$

$$K_2 = (s + R/L)I(s)|_{s=-R/L} = -\frac{1}{R}$$

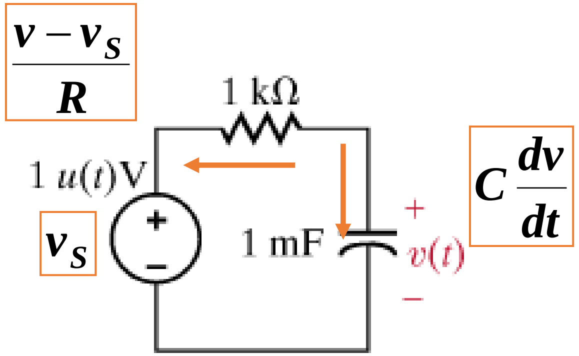
$$i(t) = \frac{1}{R}\left(1 - e^{-\frac{R}{L}t}\right); \; t > 0$$

No need to search for particular or comple-mentary solutions

GEAUX

LEARNING BY DOING

Find $v(t),\ t>0$



Model using KCL $\quad C\frac{dv}{dt} + \frac{v - v_S}{R} = 0$

$$RC\frac{dv}{dt} + v = v_S$$

$$RC\mathcal{L}\left[\frac{dv}{dt}\right] + V(s) = V_S(s)$$

$$\mathcal{L}\left[\frac{dv}{dt}\right] = sV(s) - v(0) = sV(s)$$

$$v_S(t) = 0,\ t<0 \Rightarrow v(0) = 0$$

Initial condition given in implicit form

$$v_S = u(t) \Rightarrow V_S(s) = \frac{1}{s}$$

In the Laplace domain the differential equation is now an algebraic equation

$$RCsV(s) + V(s) = \frac{1}{s}$$

$$V(s) = \frac{1}{s(RCs+1)} = \frac{1/RC}{s(s+1/RC)}$$

Use partial fractions to determine inverse

$$V(s) = \frac{1/RC}{s(s+1/RC)} = \frac{K_1}{s} + \frac{K_2}{s+1/RC}$$

$$K_1 = sV(s)|_{s=0} = 1$$

$$K_2 = (s+1/RC)V(s)|_{s=-1/RC} = -1$$

$$v(t) = 1 - e^{-\frac{t}{RC}},\ t \geq 0$$

GEAUX

CIRCUIT ELEMENT MODELS

The method used so far follows the steps:
1. Write the differential equation model
2. Use Laplace transform to convert the model to an algebraic form

For a more efficient approach:
1. Develop s-domain models for circuit elements
2. Draw the "Laplace equivalent circuit" keeping the interconnections and replacing the elements by their s-domain models
3. Analyze the Laplace equivalent circuit. All usual circuit tools are applicable and all equations are algebraic.

Independent sources

$$v_S(t) \to V_S(s)$$

$$i_S(t) \to I_S(s)$$

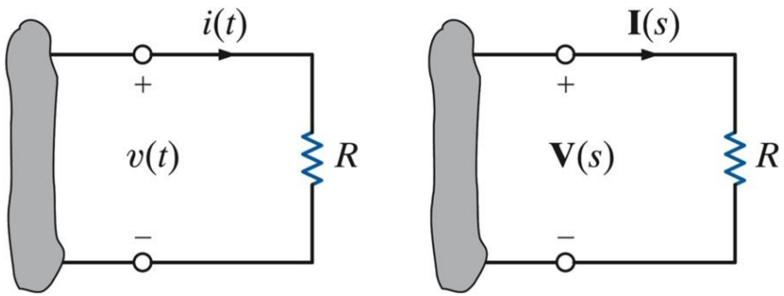
Dependent sources

$$v_D(t) = Ai_C(t) \to V_D(s) = AI_C(s)$$

$$i_D(t) = Bv_C(t) \to I_D(s) = BV_C(s)$$

...

Resistor



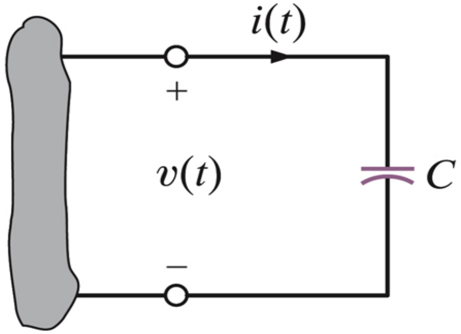
$$v(t) = Ri(t) \Rightarrow V(s) = RI(s)$$

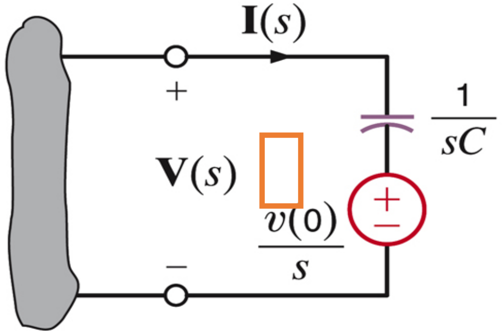
GEAUX

Capacitor: Model 1

Source transformation

$$\mathcal{L}\left[\int_0^t i(x)dx\right] = \frac{I(s)}{s}$$





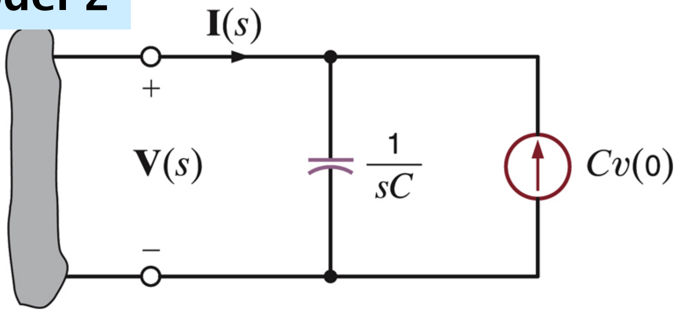
$$I_{eq} = \frac{\frac{v(0)}{s}}{\frac{1}{Cs}} = Cv(0)$$

$$v(t) = \frac{1}{C}\int_0^t i(x)dx + v(0)$$

$$V(s) = \frac{1}{Cs}I(s) + \frac{v(0)}{s}$$

Impedance in series with voltage source

Capacitor: Model 2



$$I(s) = CsV(s) - Cv(0)$$

Impedance in parallel with current source