Control Systems

Subject Code: BEC-26 Third Year ECE

Unit-III

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Lecture 5

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Previous Lecture:

Sr. No.	Input Signal	Steady State Error	Constant	Constant Expression
1	Step Input	$e_{ss}(t) = \dfrac{A}{1+K_p}$	Position Error Constant	$K_p = \lim_{s\to 0} G(s).H(s)$
2	Ramp Input	$e_{ss}(t) = \dfrac{A}{K_v}$	Velocity Error Constant	$K_v = \lim_{s\to 0} sG(s).H(s)$
3	Parabolic Input	$e_{ss}(t) = \dfrac{A}{K_a}$	Acceleration Error Constant	$K_a = \lim_{s\to 0} s^2G(s).H(s)$

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Relation between steady state error and Type of system

The type of system means the number of poles G(s)H(s) at s=0. Consider the general form,

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)..............(1+T_m s)}{s^n(1+T_a s)(1+T_b s)..............(1+T_n s)}$$

Here there are n poles at s=0. Hence the type of system is n.

Steady state error for Step input for Type 0 system

For type zero system, n=0

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots\ldots(1+T_m s)}{(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots\ldots(1+T_n s)}$$

The position error constant is given by,

$$K_p = \lim_{s\to 0} G(s).H(s)$$

$$K_p = \lim_{s\to 0} \frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots\ldots(1+T_m s)}{(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots\ldots(1+T_n s)}$$

$$K_p = \frac{K(1+T_1 0)(1+T_2 0)\ldots\ldots\ldots\ldots\ldots(1+T_m 0)}{(1+T_a 0)(1+T_b 0)\ldots\ldots\ldots\ldots\ldots(1+T_n 0)}$$

Steady state error for Step input for Type 0 system

$$K_p = \frac{K(1)(1)\ldots\ldots\ldots\ldots\ldots(1)}{(1)(1)\ldots\ldots\ldots\ldots\ldots(1)}$$

$$K_p = K$$

The steady state error is given by,

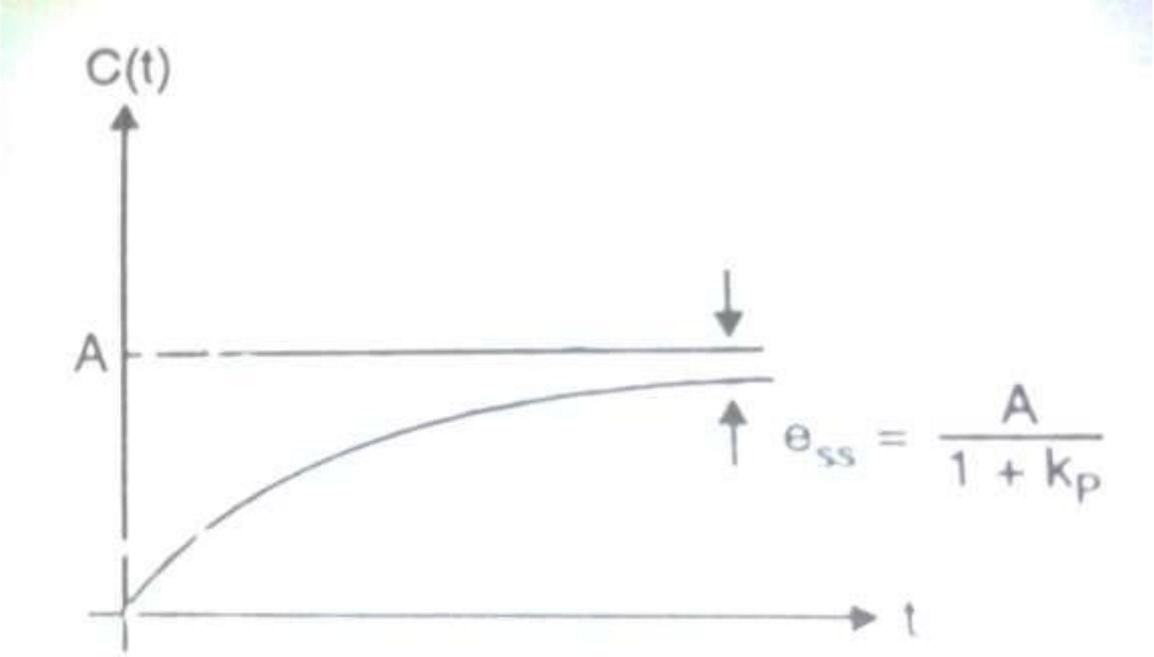
$$e_{ss}(\mathrm{t}) = \frac{A}{1+K_p}$$

$$e_{ss}(\mathrm{t}) = \frac{A}{1+K}$$

Steady state error for Step input for Type 0 system

$$e_{ss}(t) = \frac{A}{1+K}$$

A type zero system has a finite steady state error to a step input ,



Steady state error for Step input for Type 1 system

For type one system, n=1

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s(1+T_a s)(1+T_b s)................(1+T_n s)}$$

The position error constant is given by,

$$K_p = \lim_{s\to 0} G(s).H(s)$$

$$K_p = \lim_{s\to 0} \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s(1+T_a s)(1+T_b s)................(1+T_n s)}$$

$$K_p = \frac{K(1+T_1 0)(1+T_2 0)................(1+T_m 0)}{0(1+T_a 0)(1+T_b 0)................(1+T_n 0)}$$

Steady state error for Step input for Type 1 system

$$K_p = \frac{K(1)(1)\ldots\ldots\ldots\ldots\ldots(1)}{0}$$

$$K_p = \infty$$

The steady state error is given by,

$$e_{ss}(t) = \frac{A}{1 + K_p}$$

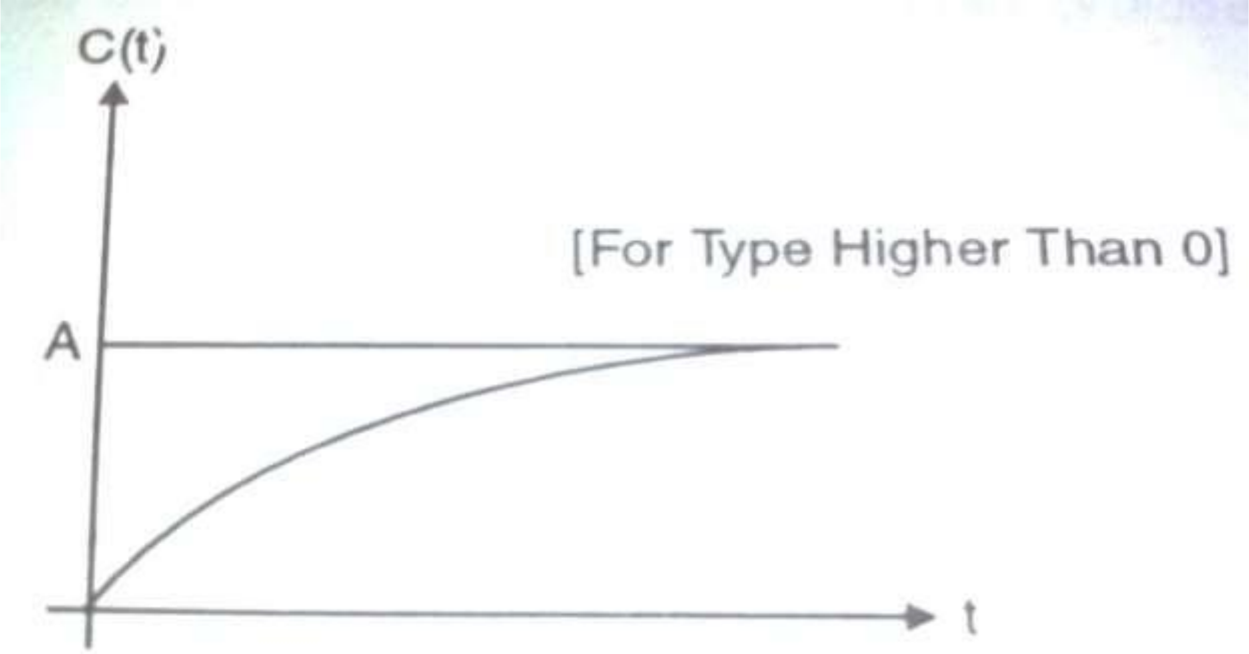
$$e_{ss}(t) = \frac{A}{1 + \infty}$$

$$e_{ss}(t) = 0$$

Steady state error for Step input for Type 1 system

$$e_{ss}(t) = 0$$

A type one system has a zero steady state error to a step input ,



Steady state error for Step input for Type 2 system

For type two system, n=2

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots\ldots(1+T_m s)}{s^2(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots\ldots(1+T_n s)}$$

The position error constant is given by,

$$K_p = \lim_{s \to 0} G(s).H(s)$$

$$K_p = \lim_{s \to 0} \frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots\ldots(1+T_m s)}{s^2(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots\ldots(1+T_n s)}$$

$$K_p = \frac{K(1+T_1 0)(1+T_2 0)\ldots\ldots\ldots\ldots\ldots(1+T_m 0)}{0(1+T_a 0)(1+T_b 0)\ldots\ldots\ldots\ldots\ldots(1+T_n 0)}$$

Steady state error for Step input for Type 1 system

$$K_p = \frac{K(1)(1)\ldots\ldots\ldots\ldots\ldots(1)}{0}$$

$$K_p = \infty$$

The steady state error is given by,

$$e_{ss}(\mathrm{t}) = \frac{A}{1+K_p}$$

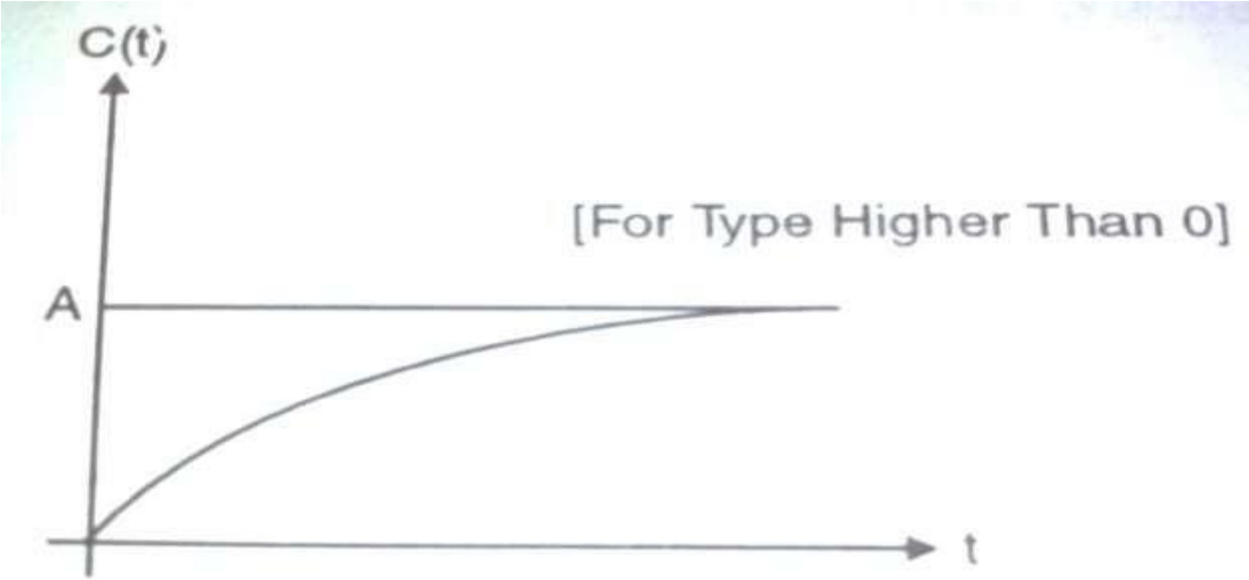
$$e_{ss}(\mathrm{t}) = \frac{A}{1+\infty}$$

$$e_{ss}(\mathrm{t}) = 0$$

Steady state error for Step input for Type 1 system

$$e_{ss}(t) = 0$$

A type two system has a zero steady state error to a step input ,



It is clear that all higher type systems except type zero have zero steady state error.

Steady state error for **Ramp input** for **Type 0** system

For type zero system, n=0

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots\ldots(1+T_m s)}{(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots\ldots(1+T_n s)}$$

The velocity error constant is given by,

$$K_v = \lim_{s\to 0} sG(s).H(s)$$

$$K_v = \lim_{s\to 0} s\left\{\frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots\ldots(1+T_m s)}{(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots\ldots(1+T_n s)}\right\}$$

$$K_v = 0\times\left\{\frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots\ldots\ldots\ldots(1+T_m s)}{(1+T_a s)(1+T_b s)\ldots\ldots\ldots\ldots\ldots(1+T_n s)}\right\}$$

Steady state error for Ramp input for Type 0 system

$$K_v = 0$$

The steady state error is given by,

$$e_{ss}(t) = \frac{A}{K_v}$$

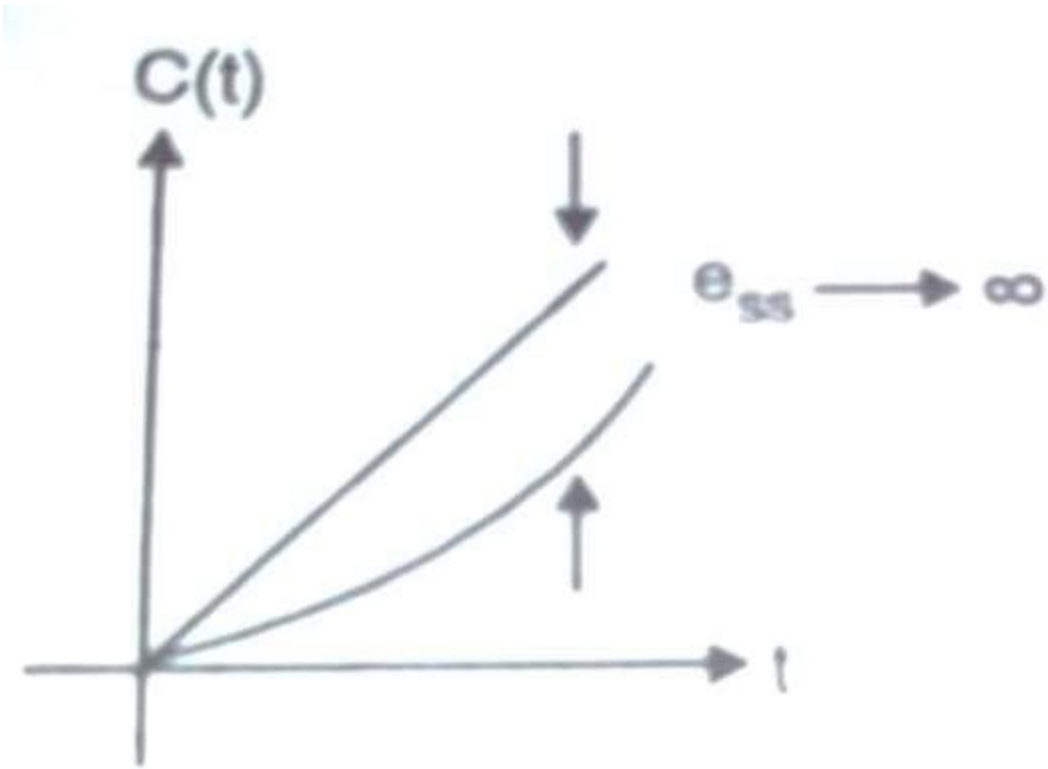
$$e_{ss}(t) = \frac{A}{0}$$

$$e_{ss}(t) = \infty$$

Steady state error for Ramp input for Type 0 system

$$e_{ss}(t) = \infty$$

The error increase continuously hence type zero system fails to track a ramp input successfully.



Steady state error for Ramp input for Type 1 system

For type one system, n=1

$$G(s).H(s) = \frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s(1+T_a s)(1+T_b s)................(1+T_n s)}$$

The velocity error constant is given by,

$$K_v = \lim_{s \to 0} sG(s).H(s)$$

$$K_v = \lim_{s \to 0} s\{\frac{K(1+T_1 s)(1+T_2 s)................(1+T_m s)}{s(1+T_a s)(1+T_b s)................(1+T_n s)}\}$$

$$K_v = \frac{K(1+T_1 0)(1+T_2 0)................(1+T_m 0)}{(1+T_a 0)(1+T_b 0)................(1+T_n 0)}$$

Steady state error for Ramp input for Type 1 system

$$K_v = K$$

The steady state error is given by,

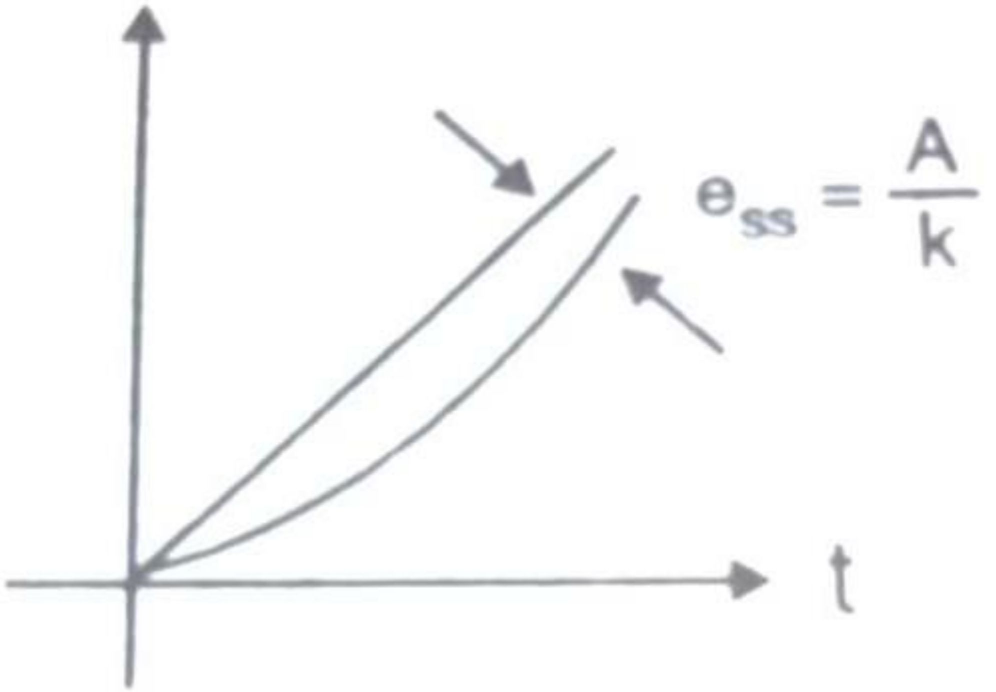
$$e_{ss}(t) = \frac{A}{K_v}$$

$$e_{ss}(t) = \frac{A}{K}$$

Steady state error for Ramp input for Type 1 system

$$e_{ss}(t) = \frac{A}{K}$$

This indicates finite steady state error for type one system for ramp input



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