# Computer Graphics (BCS-27)

# L:T:P :: 3:1:2 (Credits: 5)

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# Unit-1

- Basics of Computer Graphics
- Graphic Devices
- Simple Line Drawing Methods

# Basics of Computer Graphics

- In computer graphics, we study tools and techniques for screen control of the display unit attached with the computer.
- Screen control is done up to the level of having control over all pixel's attributes on the screen.
- Pixel refers to the smallest addressable unit of picture on the screen.
- Pixel's attributes refers to the colour and intensity of the light output from the pixel.

## Basics of Computer Graphics (contd…)

**Interactive computer graphics involves two-way communication between computer and user. The computer, upon receiving signals from the input device, can modify the displayed picture appropriately. To the user it appears that the picture is changing instantaneously in response to his commands.**

Major application areas of computer graphics includes

1. Computer Aided Design/ Drafting (CAD & CADD)

2. Presentation Graphics

3. Entertainment

4. Computer Aided Learning (CAL)

## Basics of Computer Graphics (contd…)

5. Computer Art

6. Office automation and desktop publishing

7. Graphical User-interface

8. Medical applications

9. Internet

10. Simulation and virtual reality

11. Geographical Information System (GIS)

12. Advertisement

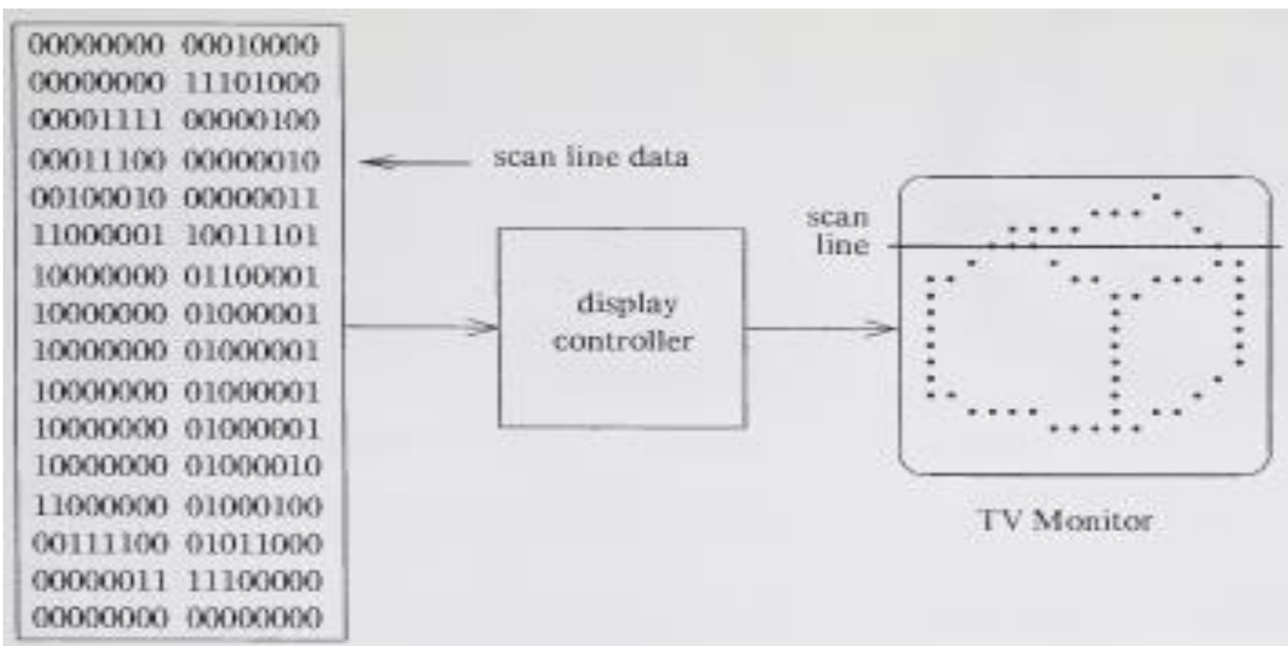
13. Simulation & Modeling

14. Architecture

15. Information Visualization and Processing

16. Image Processing

How are picture actually stored and displayed in Frame Buffer Display?



- **Difficulties for displaying pictures:** Some common questions

1. How to display straight lines & curves?

*Lines at 0, 45,and 90 degrees can be drawn easily but drawing lines at other angles or drawing curve creates problems of staircase like quantization effects that are resolved using algorithms for smoothing the lines and curves.*

2. Why is speed so important in displaying picture?

*In refresh type displays, to maintain flicker free picture, speed of refresh should not be less than 30 times per second. It becomes more relevant in interactive computer graphics.*

3. How are pictures made to grow, shrink and rotate?

*Our knowledge of how to apply such changes, or transformations, to pictures is based on standard mathematical techniques: coordinate geometry, trigonometry, and matrix methods.*

4.What happens to pictures that are too large to fit on screen?

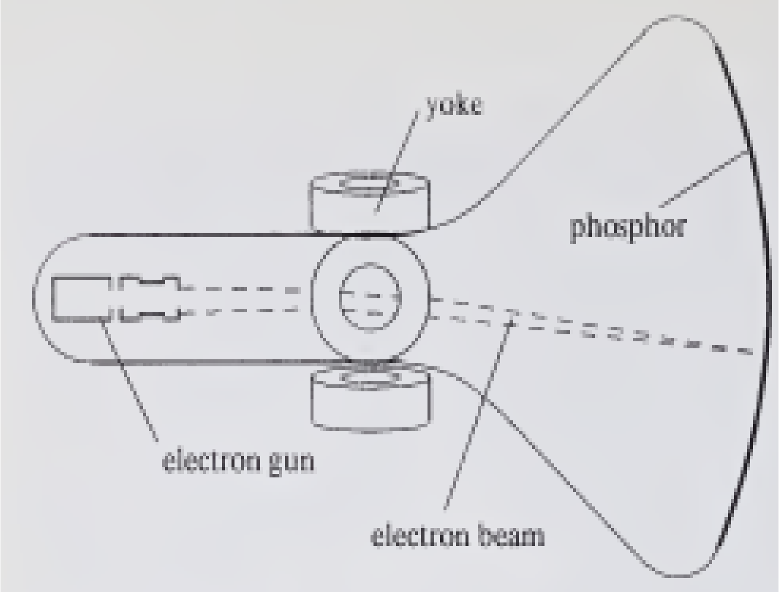
*A technique called clipping can be used to select just those parts of the picture that lie on the screen and to discard the rest. Clipping can be regarded as a special form of picture transformation, and is indeed often carried out by the same piece of software or hardware that performs other transformations.*

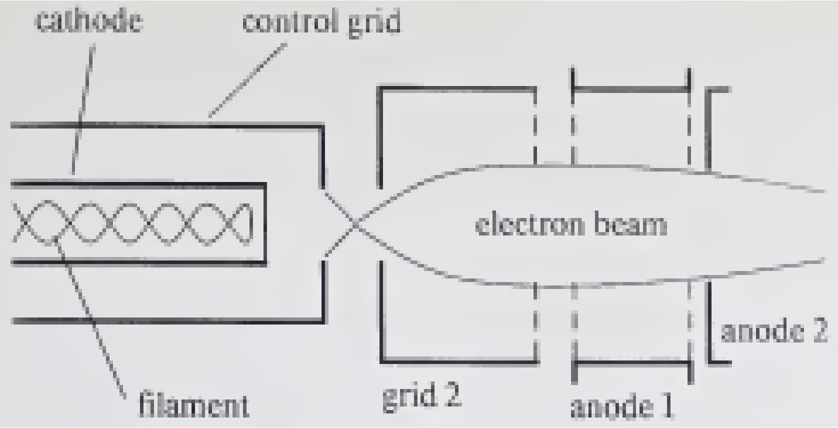
5.How can the user of display draw on the screen?

*A number of different input devices—light pen, tablet, mouse—have been invented to make this kind of interaction more convenient. In fact the computer follows every movement of the input device and is changing the picture in response to these movements.*

# Graphic Devices

When this work began in the 1950s, the Cathode Ray Tube (CRT) was the only available device capable of converting the computer's electrical signals into visible images at high speeds.




Basic Construction of the CRT

Electron Gun of the CRT

The purpose of the electron gun in the CRT is to produce an electron beam with the following properties:

1. It must be accurately focused so that it produces a sharp spot of light where it strikes the phosphor;

2. It must have high velocity, since the brightness of the image depends on the velocity of the electron beam;

3. Means must be provided to control the flow of electrons so that the intensity of the trace of the beam can be controlled.

Electrons are generated by a cathode heated by an electric filament. Surrounding the cathode is a cylindrical metal control grid, with a hole at one end that allows electrons to escape. The control grid is kept at a lower potential than the cathode, creating an electrostatic field that directs the electrons through a point source; this simplifies the subsequent focusing process. By altering the control-grid potential, we can modify the rate of flow of electrons, or beam current, and can thus control the brightness of the image; we can even cut off the flow of electrons altogether.

## Graphic Devices (contd…)

Focusing is achieved by a focusing structure containing two or more cylindrical metal plates at different potentials. These set up a toroidal electrostatic field that effectively catches straying electrons and deflects them back toward the axis of the beam. The result is a beam that is extremely finely focused and highly concentrated at the precise moment at which it strikes the phosphor. An accelerating structure is generally combined with the focusing structure. It consists of two metal plates mounted perpendicular to the beam axis with holes at their centers through which the beam can pass. The two plates are maintained at a sufficiently high relative potential to accelerate the beam to the necessary velocity; accelerating potentials of several thousand volts are not uncommon.

## Graphic Devices (contd…)

The resulting electron-gun structure has the advantage that it can be built as a single physical unit and mounted inside the CRT envelope. Other types of gun exist, whose focusing is performed by a coil mounted outside the tube; this is called electromagnetic focusing to distinguish it from the more common electrostatic method described in the preceding paragraph. The electro magnetic technique can result in finer focusing, but the electrostatic method is generally preferred in graphic displays because it leads to a cheaper gun construction.

## The Deflection System

A set of coils, or yoke, mounted at the neck of the tube, forms part of the deflection system responsible for addressing in the CRT. Two pairs of coils are used, one to control horizontal deflection, the other vertical. A primary requirement of the deflection system is that it deflect rapidly, since speed of deflection determines how much information can be displayed without flicker. To achieve fast deflection, we must use large-amplitude currents in the yoke. An important part of the deflection system is therefore the set of amplifiers that convert the small voltages received from the display controller into currents of the appropriate magnitude.

The voltages used for deflection are generated by the display controller from digital values provided by the computer. These values normally represent coordinates that are converted into voltages by digital-to-analog (D/A) conversion. To draw a vector a pair of gradually changing voltages must be generated for the horizontal and vertical deflection coils.

## Phosphors

The phosphors used in a graphic display are normally chosen for their color characteristics and persistence. Ideally the persistence, measured as the time for the brightness to drop to one-tenth of its initial value, should last about 100 milliseconds or less, allowing refresh at 30-hertz rates without noticeable smearing as the image moves. Color should preferably be white, particularly for applications where dark information appears on a light background. The phosphor should also possess a number of other attributes: small grain size for added resolution, high efficiency in terms of electric energy converted to light, and resistance to burning under prolonged excitation.

In attempts to improve performance in one or another of these respects, many different phosphors have been produced, using various compounds of calcium, cadmium, and zinc, together with traces of rare-earth elements.

**The Beam-Penetration CRT**

The normal CRT can generate images of only a single color, due to the limitations of its phosphor. A color CRT device for line-drawing displays has been developed, however; it uses a multilayer phosphor and achieves color control by modulating a normally constant parameter, namely the beam-accelerating potential.

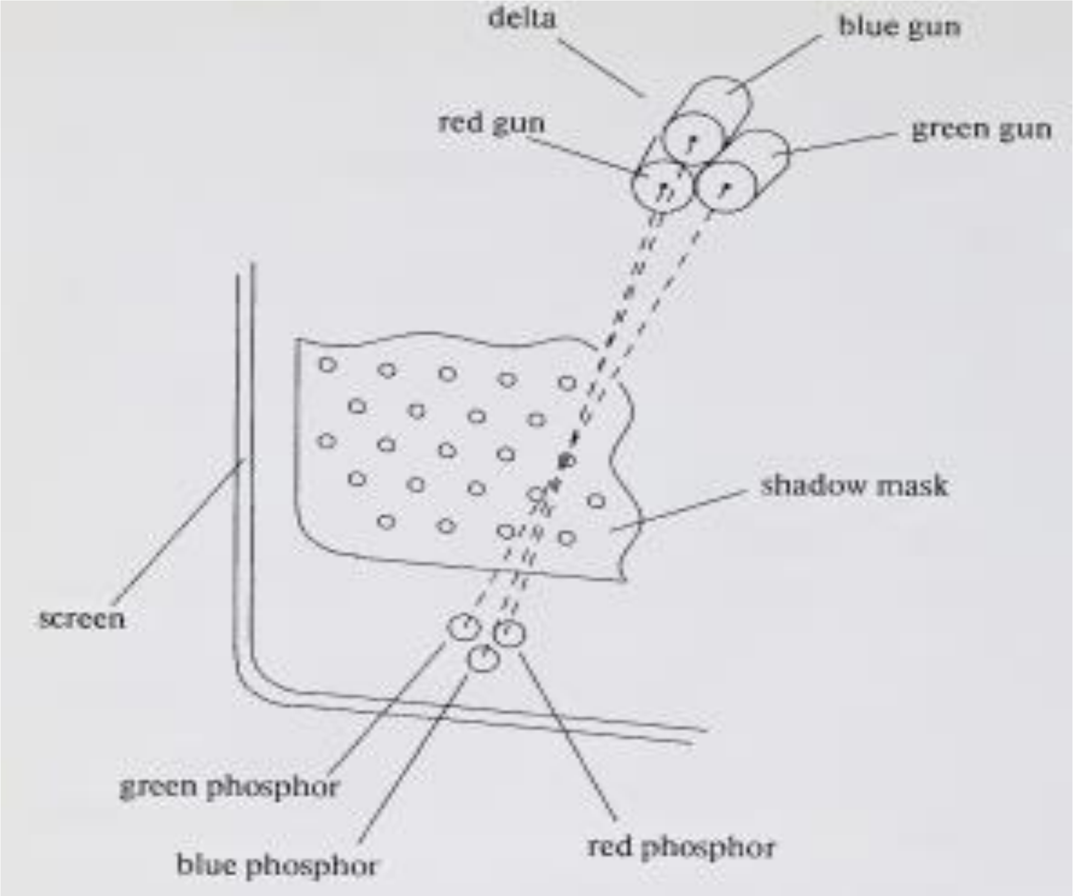
The arrangement of the beam-penetration CRT is similar to that of normal CRTs; the only unusual component is the multilayer phosphor, in which a layer of red phosphor is deposited behind the initial layer of green phosphor. If a fairly low-potential electron beam strikes the tube face, it excites only the red phosphor and therefore produces a red trace. When the accelerating potential is increased, the velocity of the beam striking the phosphor is greater, and as a result the beam penetrates into the green phosphor, increasing the green component of the light output. A limited range of colors, including red, orange, yellow and green, can be generated in this way.

## Graphic Devices (contd…)

The principal problem with the beam-penetration CRT is the need to change the beam-accelerating potential by significant amounts in order to switch colors. When the accelerating potential changes, the deflection system must react to compensate. The hardware or software must be designed to introduce adequate delays between changes in color, so that there is time for voltages to settle. In order to prevent frequent delays and consequent flicker, it is necessary to display all the red elements of the picture consecutively, then change the accelerating potential and display the yellow elements, and so on through all the different colors.

## The Shadow-Mask CRT

The shadow-mask color CRT can display a much wider range of colors than the beam penetration CRT, and is used in the majority of color TV sets and monitors



## Graphic Devices (contd…)

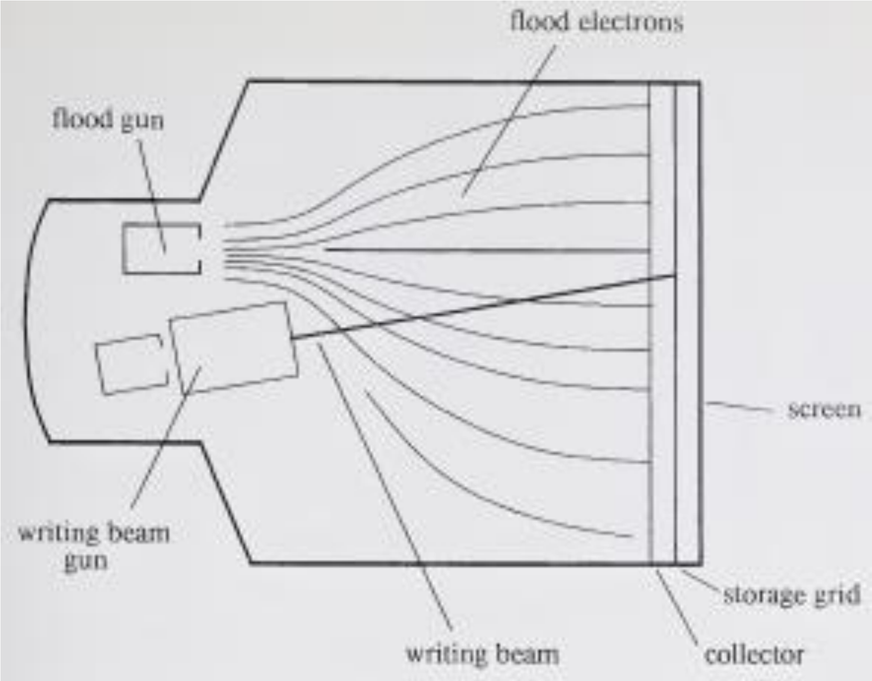
Just behind the phosphor- coated face of the CRT is a metal plate, the shadow mask, pierced with small round holes in a triangular pattern. In place of the usual single electron gun, the shadow-mask tube uses three guns, grouped in a triangle or delta. These three guns are responsible for the red, green, and blue components of the light output of the CRT. The deflection system of the CRT operates on all three electron beams simultaneously, bringing all three to the same point of focus on the shadow mask. Where the three beams encounter holes in the mask, they pass through and strike the phosphor. Since they originate at three different points, however, they strike the phosphor in three slightly different spots. The phosphor of the shadow-mask tube is therefore laid down very carefully in groups of three spots—one red, one green, and one blue—under each hole in the mask, in such a way that each spot is struck only by electrons from the appropriate gun. The effect of the mask is thus to "shadow" the spots of red phosphor from all but the red beam, and likewise for the green and blue phosphor spots. We can therefore control the light output in each of the three component colors by modulating the beam current of the corresponding gun.

remained relatively expensive compared with the monochrome CRT, and still has a relatively poor performance in all respects except color range. The shadow-mask CRT compares particularly unfavorably in resolution and in efficiency of light output. Both these effects are caused by the use of the shadow mask: the grain of the triangular pattern of holes sets a limit on attainable resolution, and the mask tends to block a large proportion of the available beam energy, reducing the total brightness. With the use of very high accelerating potentials it is, however, possible to match the brightness of monochrome CRT images.

A further, unique problem with the shadow-mask tube is that of convergence. It is extremely difficult to adjust the three guns and the deflection system so that the electron beams are deflected exactly together, all three converging on the same hole in the shadow mask. Where they fail to converge the three component colors appear to spread in a manner reminiscent of a poorly-aligned color printing process. Often it is possible to achieve adequate convergence over only a limited area of the screen.

## Direct-view Storage Tube (DVST)

Outwardly the DVST behaves like a CRT with an extremely long-persistence phosphor. A line written on the screen will remain visible for up to an hour before it fades from sight. Inwardly, too, the DVST resembles the CRT, since it uses a similar electron flood gun and a somewhat similar phosphor-coated screen. The general arrangement of the DVST is shown in Figure given below.



## Graphic Devices (contd…)

The beam is designed not to write directly on the phosphor, however, but on a fine-mesh wire grid, coated with dielectric and mounted just behind the screen. A pattern of positive charge is deposited on the grid, and this pattern is transferred to the phosphor by a continuous flood of electrons issuing from a separate flood gun. Just behind the storage mesh is a second grid, the collector, whose main purpose is to smooth out the flow of flood electrons. These electrons pass through the collector at a low velocity, and are attracted to the positively charged portions of the storage mesh but repelled by the rest. Electrons not repelled by the storage mesh pass right through it and strike the phosphor. In order to increase the energy of these relatively slow-moving electrons and thus create a bright picture, the screen is maintained at a high positive potential by means of a voltage applied to a thin aluminum coating between the tube face and the phosphor.

Until they pass through the mesh, the flood electrons are still moving fairly slowly and therefore hardly affect the charge on the mesh. One of the problems with the DVST is in fact the difficulty in removing the stored charge to erase the picture.

## Graphic Devices (contd…)

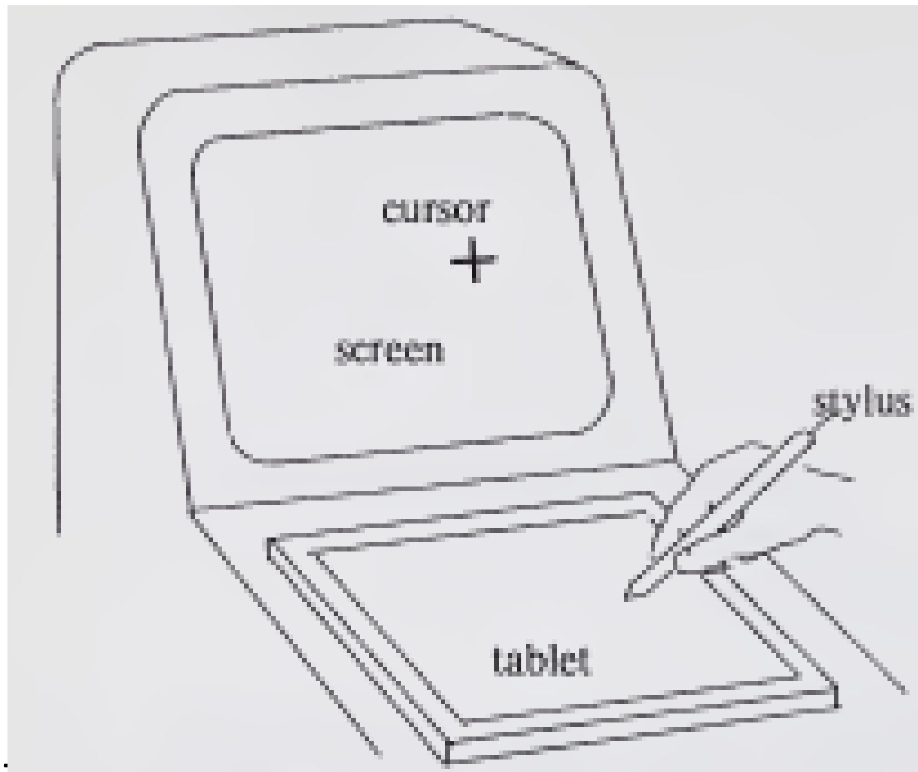
The normal erasing method is to apply a positive voltage to the storage mesh for one second or more; this removes all the charge but also generates a rather unpleasant flash over the entire screen surface. This erase problem is perhaps the most severe drawback of the DVST, for it prevents the use of the device for dynamic graphics applications. Other problems are its relatively poor contrast, a result of the comparatively low accelerating potential applied to the flood electrons, and the gradual degradation of the picture quality as background glow accumulates; this glow is caused by the small amounts of charge deposited on the mesh by repelled flood electrons.

In terms of performance, the DVST is somewhat inferior to the refresh CRT. Only a single level of line intensity can be displayed, and only green-phosphor tubes are available. Until recently, the DVST used relatively small-screen tubes; now tubes with 19-inch and 25-inch diagonals are available. The smaller DVSTs have the advantage of a flat screen, not present in the larger variety. Some storage-tube displays possess the capability to refresh a limited number of vectors.

## Graphic Devices (contd…)

**TABLETS**

The term tablet is used to describe a flat surface, generally separate from the display, on which the user draws with a stylus (see Figure given below).




The similarity of the tablet and stylus to paper and pencil makes them a particularly natural combination for graphical input.

## Early Tablet Devices

The RAND Tablet was developed at the RAND Corporation. It provides a flat drawing area 10 inches square and rests on a table top. Embedded in the surface of the tablet are 1024 lines parallel to the x axis and 1024 lines parallel to the y axis. Each individual line carries a unique digitally coded signal that can be picked up by the stylus. Inside the stylus a sensitive amplifier detects the pulses from the lines, amplifies them, and delivers them via coaxial cable to decoding logic, which in turn deposits binary integer coordinates in the tablet's buffer registers. The stylus has a small switch in the tip, whose status is kept in an extra bit in the buffer register.

An alternative coordinate-input technique uses voltage gradients within a resistive plate. In the simplest configuration, a sheet of partially conductive material is used as the tablet surface. During successive time intervals, a potential is applied first horizontally and then vertically across the sheet. The stylus is kept in contact with the conductive sheet and senses a potential corresponding to its position.

## Graphic Devices (contd…)

The x and y coordinates of the pen can be determined by measuring the potential during the horizontal and vertical time periods. Absence of any potential indicates that the pen is not in contact with the surface.

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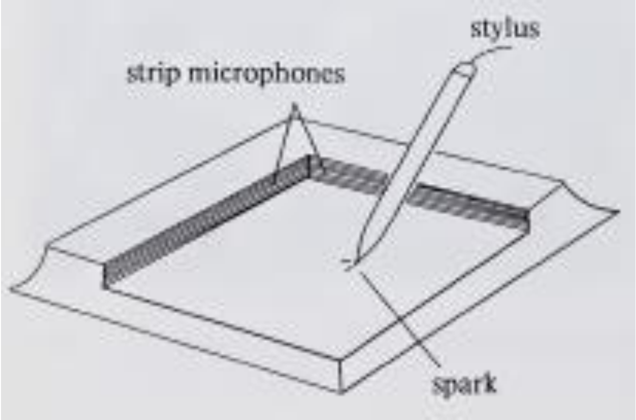
## The Acoustic Tablet

It depends on the use of strip microphones, which are mounted along two adjacent edges of the tablet, as shown in Figure given below.




The stylus has a small piece of ceramic mounted close to its tip, and at regular intervals a small spark is generated across the surface of the ceramic between two electrodes. The microphones pick up the pulse of sound generated by the spark, and two counters record the delay between creating the spark and receiving the sound. These two delays are proportional to the stylus distance from the two edges of the tablet where the microphones are mounted. They can therefore be used as x and y values.

**The Electro-acoustic Tablet**

Another acoustic technique has been employed in the electro-acoustic tablet: in this device, the writing surface is a sheet of magnetostrictive material acting like a row of delay lines. An electric pulse travels through the sheet, first horizontally and then vertically, and is detected by a sensor in the stylus as it passes by. A counter is used to determine the delay from the time the pulse is issued to the time it is detected; from this value the position of the stylus can be determined. Pulses may be issued at any frequency up to about 200 pairs per second, adequate to track the stylus at 5-millisecond intervals.

The electro-acoustic tablet is quieter in operation than its acoustic counterpart and is less affected by ambient noise or air movement. Both types of tablet can be constructed to sizes in excess of 1 meter square.

**Light Pen** is a pointing device. If it is pointed at an item on the screen, it generates information from which the item can be identified by the program. However, the light pen does not generally have any associated tracking hardware. Instead tracking is performed by software, making use of the output function of the display.

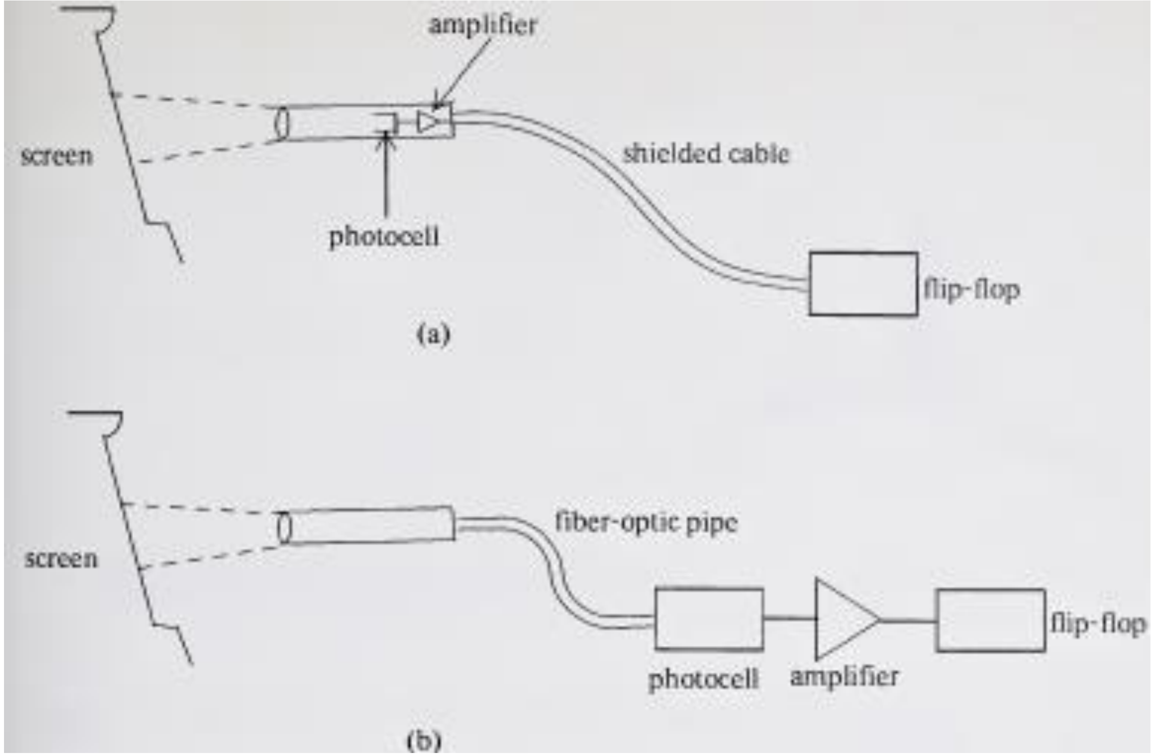



*Figure: The light pen: (a) using a hand-held photocell; (b) using a fiber-optic pipe.*

## Graphic Devices (contd…)

Two alternative arrangements are shown in Figure. In each case the two main elements of the light pen are a photocell and an optical system which focuses onto it any light in the pen's field of view. A pen-shaped housing permits the light pen to be held in the hand and pointed at the display screen. On this housing is either a finger- operated switch or a shutter that must be depressed to allow light to reach the photocell. The output of the photocell is amplified and fed to a flip-flop which is set whenever the pen is pointed at a sufficiently bright source of light. This flip-flop can be read and cleared by the computer.

Pointing operations are easily programmed for the light pen, particularly if we are using a point-plotting display: we can test the light-pen flip-flop after displaying each point and thus determine the exact spot at which the pen is pointing. Alternatively we can use an interrupt feature to indicate when the flip-flop is set. The computer can read the contents of the display's address register when an interrupt occurs and from them determine which item was seen by the pen.

## Graphic Devices (contd…)

All light-pen programs depend on a rapid response from the pen when it is pointed at the screen. A particularly fast response is required if the light pen is to be used with high-speed displays. Suppose, for example, a display executes one instruction every 2 microseconds but the delay between displaying a point or line and setting the light pen flip-flop is 3 microseconds. By the time this happens, the display will be processing either the next instruction or the one after that. The program may therefore incorrectly identify the seen item.

Fast-response light pens can be built by using a highly sensitive photocell such as a photomultiplier tube. However, this sort of device is too bulky to be held in the hand, so the light must be focused onto it by a fiber-optic pipe, as shown in Figure b. Transistor-type photocells, such as the photodiode, are cheap and small enough to be hand-held. However, photodiodes generally take one or more microseconds to respond and are therefore more suited to light pens for slower displays.

## Graphic Devices (contd…)

The light pen communicates with the computer through a single 1-bit status register that is set whenever the light pen sees an intensified spot on the screen. The identity of the spot is determined by reading the display address register as soon as the status bit is set. Instead of relying on the computer to respond immediately, the display processor halts when the status bit is set, thus ensuring that the address register will still be valid when the computer reads it. Nevertheless it is important for the computer to respond quickly to a change in the status bit; if the display remains off for more than a millisecond or two, the user will notice a flickering effect. The status bit generates an interrupt when it is set, and this invokes a high-priority task that reads the display address register and restarts the display.

Two kinds of light-pen interrupts may occur. The user may point the pen at an item on the screen in order to select it; this results in a selection interrupt. When the user is positioning with the pen, a tracking pattern is displayed in order to follow the pen's movement and tracking interrupts are generated when the pen sees the pattern. The light-pen task must distinguish between selection and tracking interrupts.

## Graphic Devices (contd…)

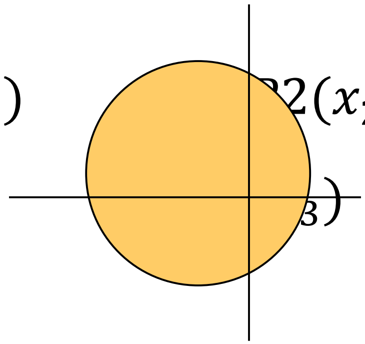
This can be achieved by checking the display address register to see whether the display has been stopped while displaying the tracking pattern. If it has, the polling task proceeds with the tracking process; if it has not, the interrupt is treated as a selection interrupt and a selection event is generated.

Light-Pen Tracking

When the light pen is used for positioning, a tracking pattern is displayed in the vicinity of the pen’s field of view. When the pattern is seen by the pen a tracking interrupt is generated, from which the position of the pen is determined. If the pen has moved since the previous interrupt, the tracking pattern is moved to the new pen position. Thus the pattern follows the pen’s movement around the screen. *Centre of field of view is the point* $P2(x_4, y_4)$

$$\left(\frac{(x_1+x_2)}{2}, \frac{(y_3+y_4)}{2}\right)$$

P1$(x_1, y_1)$



## Graphic Devices (contd…)

To achieve accurate tracking a cross shaped pattern should be used. Arms of the cross are drawn inward, starting at the extremity of each arm. They are drawn point by point so that the light pen will interrupt at the moment each arm of the cross emerges into the field of the view. If the coordinates of the most recent points are read after each interrupt, the four points $P_1, P_2, P_3, P_4$ will be found which lie on the circumference of the field of view. Centre of field of view is the point

$$\left(\frac{(x_1+x_2)}{2}, \frac{(y_3+y_4)}{2}\right)$$

# Simple Line Drawing Methods

## Point-plotting Techniques

The very first graphical displays were of the point-plotting variety. They did not use frame buffers but were fed with a stream of point coordinates by the computer. Only a very limited number of points could be displayed in this fashion without flicker.

Point-plotting displays of this kind were made obsolete by the introduction of line-drawing displays in the mid-1960s. The line-drawing display can draw complete segments of straight lines without plotting each individual pixel on the line; it therefore has a much higher capacity than the point-plotting display for line drawings. It also does away with the need to compute the position of each pixel in the picture.

Despite the obsolescence of the original point-plotting displays, the techniques developed for programming them remain relevant today. The main reason is that point-plotting techniques have become essential in programming frame-buffer displays, where once again the intensity of each dot must be separately computed. Point-plotting techniques also serve to introduce us to the incremental methods so frequently useful in computer graphics.

## Simple Line Drawing Methods (contd…)

### COORDINATE SYSTEMS

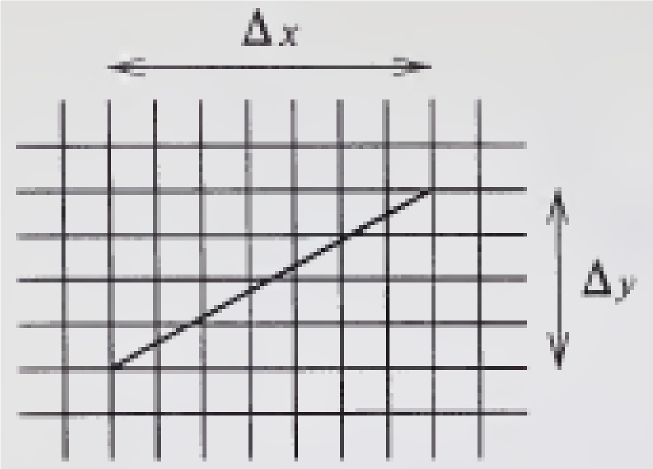
Point-plotting techniques are based on the use of a cartesian coordinate system. Points are addressed by their x and y coordinates; the value of x increases from left to right and y likewise from bottom to top.

Points are plotted in response to digital signals from the computer. This means that they cannot be positioned with infinite precision; instead we are limited by the precision of the digital values presented to the display.

In most cases precision is based on the resolution of the display screen. Nothing is gained by increasing coordinate precision much beyond the resolution of the screen because the observer will not be able to tell the difference. If precision is much less than resolution, however, there will be resolvable points on the screen at which it is impossible to display a dot; this will cause visible gaps in lines. Hence when a display is designed, its coordinate precision is made approximately equal to screen resolution.

**Qualities of good line drawing algorithms:**

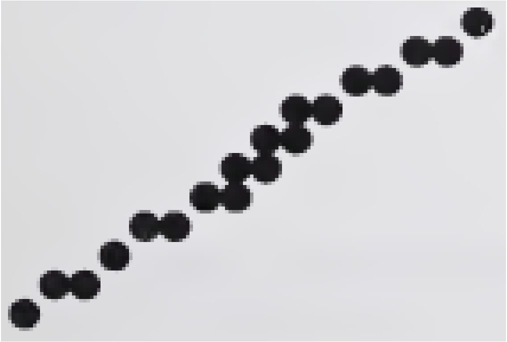
*1.Lines should appear straight.* Point-plotting techniques are admirably suited to the generation of lines parallel or at 45° to the x and y axes. Other lines cause a problem: a line segment, though it starts and finishes at addressable points, may happen to pass through no other addressable points in between, figure (a) shows such a line. In these cases we must approximate the line by choosing addressable points close to it. If we choose well, the line will appear straight; if not, we shall produce crooked lines, as in Figure (b).




## Simple Line Drawing Methods (contd…)

*2. Lines should terminate accurately*. Unless lines are plotted accurately, they may terminate at the wrong place. The effect is often seen as a small gap between the endpoint of one line and the starting point of the next or as a cumulative error.

*3. Lines should have constant density*. With bright lines plotted on a dark background, line density is observed as brightness; when the line is black and the background light, it is seen as blackness. In either case, line density is proportional to the number of dots displayed divided by the length of the line. To maintain constant density, dots should be equally spaced. This can be achieved only in lines parallel or at 45° to the axes. In other cases, we must attempt to achieve as even spacing as possible; bunching of dots will otherwise be visible as particularly bright or dark regions on the line (see the example Figure given below)



Uneven line density caused by bunching of dots.

## Simple Line Drawing Methods (contd…)

*Line density should be independent of line length and angle*. This is a difficult requirement to satisfy. As we have just seen, to achieve constant line density we must maintain a constant number of dots per unit length. Before plotting the line we must therefore determine its exact length, which involves computing a square root. Also we must be able to control the rate, in terms of distance traveled, at which dots are plotted. Neither of these is easily done. Normally the best we can do is to compute an approximate line-length estimate and to use a line-generation algorithm that keeps line density constant to within the accuracy of this estimate.

*Lines should be drawn rapidly*. In interactive applications we would like lines to appear rapidly on the screen. This implies using the minimum of computation to draw the line; ideally, this computation should be performed by special-purpose hardware.

## Digital Differential Analyzer (DDA)

The Symmetrical DDA

we can build DDAs to draw curves as well as straight lines provided these curves can be defined by ordinary differential equations.

Equation of a straight line is given by $y = mx + c$,

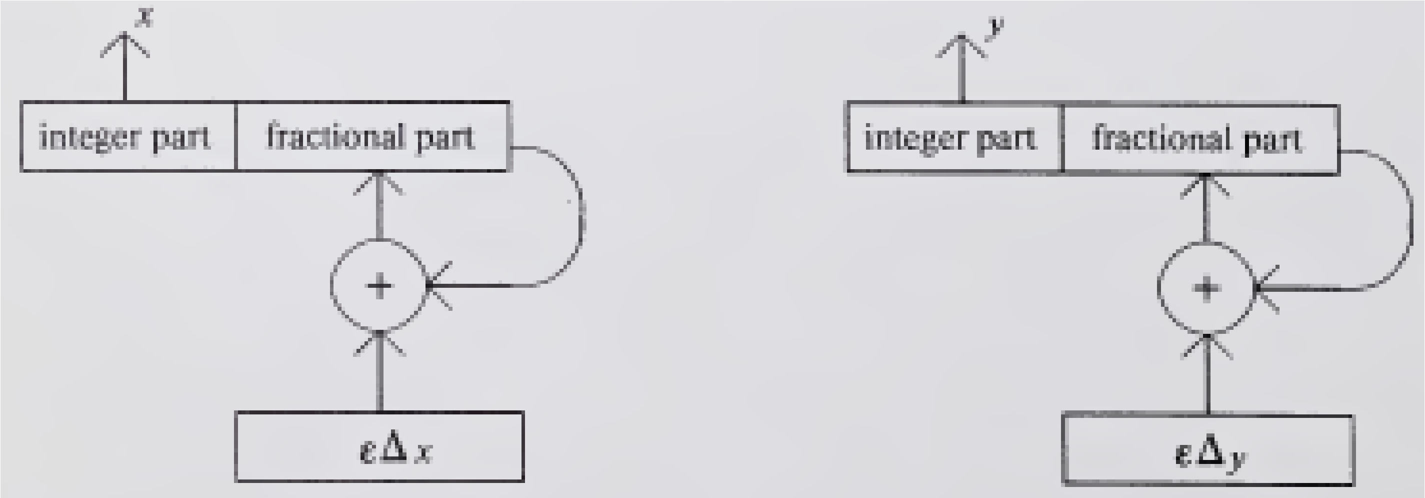
Where $m = \frac{\Delta y}{\Delta x}$, $\Delta y = (y_2 - y_1)$ and $\Delta x = (x_2 - x_1)$ is gradient of line joining the points $(x_1, y_1)\ and (x_2, y_2)$.

On differentiating the line equation we have $dy = mdx + 0$, or $\frac{dy}{dx} = m$

Thus the equation of a straight line can be defined by $\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$ which is an ordinary differential equation, hence we can build DDA for drawing straight line.

$\frac{dy}{dx} = \frac{\Delta y}{\Delta x} \Rightarrow$ ($dy \propto \Delta y$ and $dx \propto \Delta x$) or ($dy = \varepsilon \Delta y$ and $dx = \varepsilon \Delta x$)

Where $\varepsilon = 2^{-n}$, and ($2^{(n-1)} \leq \max(|\Delta x|, |\Delta y|) < 2^n$)




- The incrementing values are repeatedly added to the fractional parts, and whenever the result overflows, the corresponding integer part is incremented. The integer parts of the x and y registers are used in plotting the line. This would normally have the effect of truncating rather than rounding, so we initialize the DDA with the value 0.5 in each of the fractional parts to achieve true rounding.
- One advantage of this arrangement is that it allows us to detect changes in x and y and hence to avoid plotting the same point twice.
- This iteration continues till $x_2$ is reached If $(|\Delta x| > |\Delta y|)$ else till $y_2$ .

Example: Using symmetrical DDA to draw line joining points (2,4) & (5,9).

1. Compute $\Delta x = (x_2 - x_1) = 5 - 2 = 3$ and $\Delta y = (y_2 - y_1) = (9 - 4) = 5$
2. $(2^{(n-1)} \le \max(|\Delta x|, |\Delta y|) < 2^n)$=$(2^2 \le \max(3,5) < 2^3) \Rightarrow n = 3$
3. $\varepsilon = 2^{-n} = 2^{-3} = .125$
4. x-increment $dx = \varepsilon \Delta x = .125 \times 3 = .375$,
5. y-increments $dy = \varepsilon \Delta y = .125 \times 5 = .625$
6. Plot the initial point (2,4) and initialize the DDA registers.

| SN | x-int | x-fra | x-inc | X-New | y-int | y-fra | y-inc | Y-New | Plot |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 0.5 | 0.375 | 2.875 | 4 | 0.5 | 0.625 | 5.125 | (2,5) |
| 2 | 2 | 0.875 | 0.375 | 3.25 | 5 | 0.125 | 0.625 | 5.75 | (3,5) |
| 3 | 3 | 0.25 | 0.375 | 3.625 | 5 | 0.75 | 0.625 | 6.375 | (3,6) |
| 4 | 3 | 0.625 | 0.375 | 4 | 6 | 0.375 | 0.625 | 7 | (4,7) |
| 5 | 4 | 0 | 0.375 | 4.375 | 7 | 0 | 0.625 | 7.625 | (4,7) |
| 6 | 4 | 0.375 | 0.375 | 4.75 | 7 | 0.625 | 0.625 | 8.25 | (4,8) |
| 7 | 4 | 0.75 | 0.375 | 5.125 | 8 | 0.25 | 0.625 | 8.875 | (5,8) |
| 8 | 5 | 0.125 | 0.375 | 5.5 | 8 | 0.875 | 0.625 | 9.5 | (5,9) |

## Simple Line Drawing Methods (contd...)

Simple DDA is a simplification over Symmetrical DDA where one of the two arrangements for computation of incremental values of (x,y) is replaced with a counter that corresponds to $\max(|\Delta x|, |\Delta y|)$.

```
procedure DDA (x1, y1, x2, y2: integer);
•  var length, i: integer; x, y, xincrement, yincrement: real;
•  begin
   length= abs(x2 — x1);
   if abs(y2 — y1)> length then length = abs{y2 — y1);
   xincrement = (x2 — x1)/length; yincrement = (y2 — y1)/length;
   x = x1 + 0.5; y= y1 + 0.5;
   for i=1 to length do
      begin
      Plot(trunc(x), trunc(y));
      x =x + xincrement;  y= y + yincrement;
      end
•  end;
```

Example: Using Simple DDA to draw line joining points (2,4) & (5,9).

1. Compute $\Delta x = (x_2 - x_1) = 5 - 2 = 3$ and $\Delta y = (y_2 - y_1) = (9 - 4) = 5$
2. length=max(3,5)=5, xincrement=3/5=0.6; yincrement=5/5=1;
3. x=x1+0.5; y=y1+0.5,
4. For i=1 to length begin  Plot (trunc(x), trunc(y));

   x=x+xincrement; y=y+yincrement;

   end;

5.Stop.

| SN | x | x-inc | X-New | y | y-inc | Y-New | Plot |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 0.5 | 2.5 | 4 | 0.5 | 4.5 | (2,4) |
| 2 | 2.5 | 0.6 | 3.1 | 4.5 | 1 | 5.5 | (3,5) |
| 3 | 3.1 | 0.6 | 3.7 | 5.5 | 1 | 6.5 | (3,6) |
| 4 | 3.7 | 0.6 | 4.3 | 6.5 | 1 | 7.5 | (4,7) |
| 5 | 4.3 | 0.6 | 4.9 | 7.5 | 1 | 8.5 | (4,8) |
| 6 | 4.9 | 0.6 | 5.5 | 8.5 | 1 | 9.5 | (5,9) |

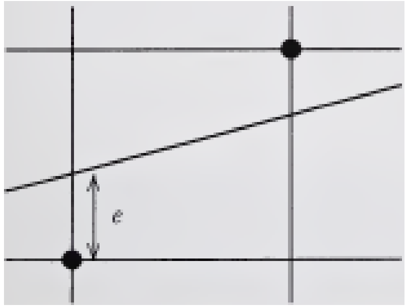
## Bresenham's Algorithm

It is designed so that each iteration changes one of the coordinate values by ±1. The other coordinate may or may not change, depending on the value of an error term maintained by the algorithm. This error term records the distance, measured perpendicular to the axis of greatest movement, between the exact path of the line and the actual dots generated.

At each iteration of the algorithm the slope of the line,

A $\frac{\Delta y}{\Delta x}$ , is added to the error term e. Before this is done,

the sign of e is used to determine whether to increment




the y coordinate of the current point. A positive e value indicates that the exact path of the line lies above the current point; therefore the y coordinate is incremented, and 1 is subtracted from e. If e is negative the y coordinate value is left unchanged.

## Simple Line Drawing Methods (contd…)

Thus the basic Bresenham’s algorithm is expressed as follows

{ Note: e is real; x, y, deltax, deltay are integers }

- e: = (deltay/deltax) — 0.5;
- for i= 1 to deltax do begin
- Plot(x, y)
- if e > 0 then begin
- y: = y + i;
- e = e -1
- end;
- x : =x + 1;
- e: = e + (deltay/deltax)
- end;

The Circle-generating DDA

The principle of the DDA can be extended to other curves; one such curve is the circular arc. The differential equation of a circle with center at the origin can be written

$\frac{dy}{dx} = \frac{-x}{y} \Rightarrow (dy \propto -x$ and $dx \propto y)$ or $(dy = -\varepsilon x$ and $dx = \varepsilon y)$

$x_{n+1} = x_n + \varepsilon y_n,\ y_{n+1} = y_n - \varepsilon x_n$ …….(1)

where $\varepsilon = 2^{-n}$, and $(2^{(n-1)} \leq r < 2^n)$

r being the radius of the circle.

Unfortunately the method just described plots a spiral, not a circular arc. Each step is made in a direction perpendicular to a radius of the circle; each point is therefore slightly farther from the center than the one before. This problem is easily solved by following modifications:

$x_{n+1} = x_n + \varepsilon y_n,\ y_{n+1} = y_n - \varepsilon x_{n+1}$ ……..(2)

Solution is based on following reasoning

## Simple Line Drawing Methods (contd…)

Equations (1) can be written in matrix form as

$$[x_{n+1} \quad y_{n+1}] = [x_n \quad y_n] \begin{bmatrix} 1 & -\varepsilon \\ \varepsilon & 1 \end{bmatrix}$$

The determinant of the matrix on the right does not equal unity but 1 + e2; this implies that the curve will spiral out. If the determinant can be reduced to unity, the curve will close. We achieve this effect by modifying the matrix as follows:

$$[x_{n+1} \quad y_{n+1}] = [x_n \quad y_n] \begin{bmatrix} 1 & -\varepsilon \\ \varepsilon & 1-\varepsilon^2 \end{bmatrix}$$