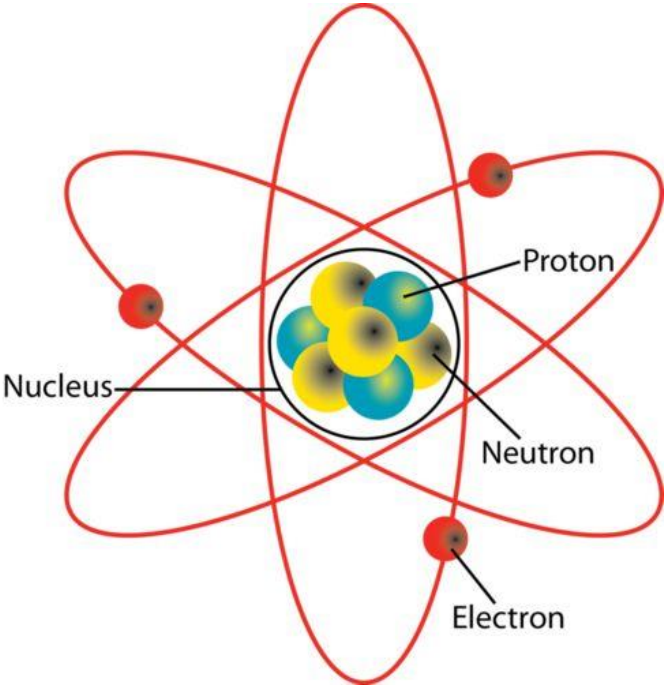
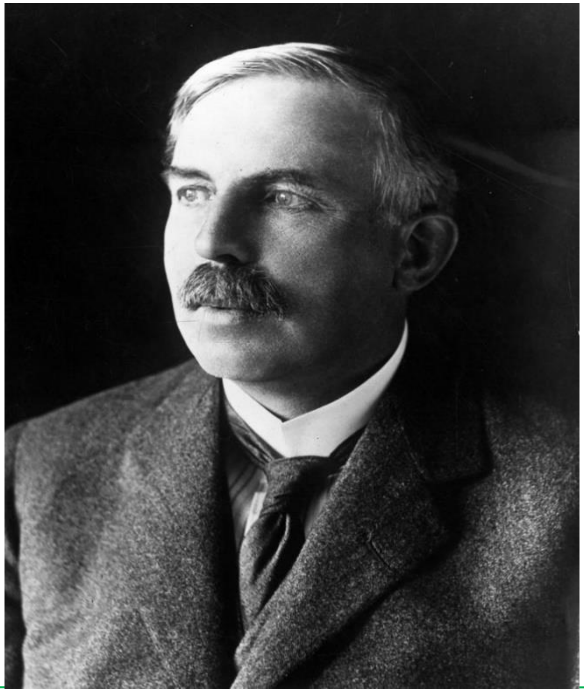
MPM: 203 NUCLEAR AND PARTICLE PHYSICS

UNIT –I: Nuclei And Its Properties

Lecture-3

By Prof. B. K. Pandey, Dept. of Physics and Material Science





Scattering Cross Section

- During the scattering experiment a narrow collimated beam of mono energetic α- particles is allowed to fall on a very thin foil of metal such gold (Z=79) or silver (Z=47) of the size 10^{-7} m or less

- The α- particles do not lose any appreciable energy in passing through such thin foils.

- The arrangement was made to count the number of α-particles scattered in different directions.

Scattering Cross Section

- The force experience by the α- particles is a central force varying inversely as the square of its distance from the nucleus.

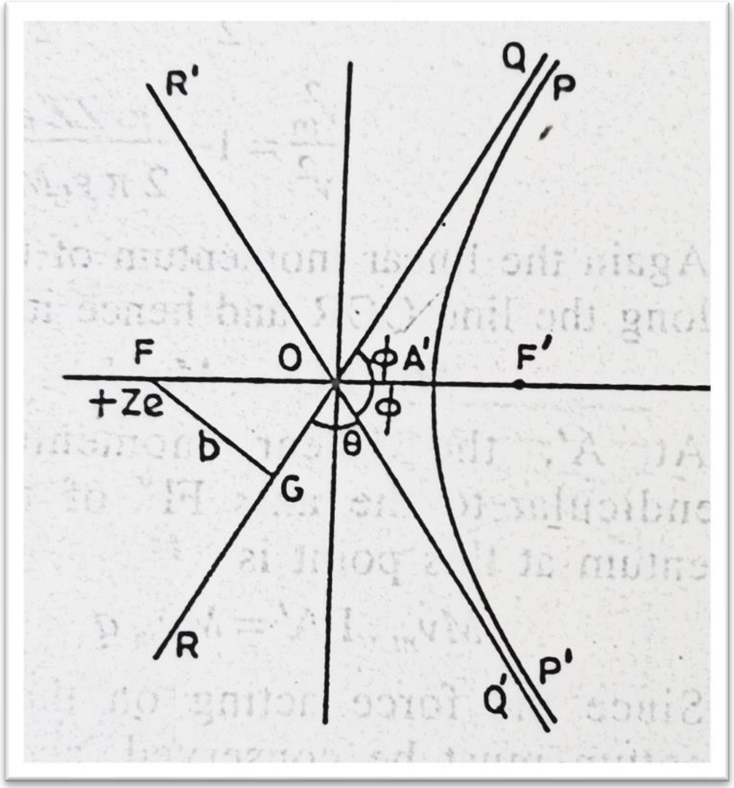
- It is known from the the of mechanics that the trajectory of a particle acted upon by such a force is conic, depending on their initial energy it may be ellipse (E<0), parabola (E=0) and hyperbola (E>0).

- In the present case initial energy of α- particles is positive (E>0) so the trajectory PA'P' is the hyperbola with the scattering nucleus located at one of its foci F.

Scattering Cross Section

- Here we assume that the entire positive charge +Ze of the nucleus is concentrated at the point F.

- The PA'P' is the trajectory of α-particle under the action of electrostatic repulsion of nucleus of charge +Ze.



- Let M and Z'e be the mass and charge of the α- particles: here Z' =2, the nucleus is assumed to be infinitely heavy.

Scattering Cross section

- If the α- particle is far away from the nucleus the repulsive force on α- particle is negligible and this particle will follow the path of straight line coinciding the with the asymptote QOR of the parabola PA'P'.
- $E_\alpha = \frac{1}{2}mv^2$ where v is the initial velocity of the α- particle.
- A perpendicular from F to G from nucleus to asymptote i.e. FG=b is known as impact parameter.
- As α- particle approaches the nucleus its trajectory bends more and more away from the F because of increasing electrostatic repulsion.
- Finally when it reaches the point A' at a minimum distance from F, it begins to move away from the nucleus along A'P'.

Scattering Cross Section

- At a great distance from the nucleus, its path of recession coincides with the asymptote R'OQ' of the hyperbola. The ange ROQ' between the two asymptotes QOR and R'OQ' is the angle of the scattering θ.

- The two asymptotes are equally inclined at an angle $\boldsymbol{\phi}$ to the axis FF' of the hyperbola.

- When the α- particle reaches at the vertex A' of the parabola the electrostatic repulsion on it is maximum.

- For the least distance of approach FA' = q, the potential energy of the α- particle at A' is given as

Scattering Cross Section

- V = $\frac{ZZ'e^2}{4\pi\varepsilon_0 q}$
- where ε_0 is the dielectric constant of the empty space

- The velocity v_m of the α- particle at A' is the minimum. Its kinetic energy at this point is $\frac{1}{2}mv_m^2$ so the total energy of this particle is given as
- E= E_k +V = $\frac{1}{2}mv_m^2$ + $\frac{ZZ'e^2}{4\pi\varepsilon_0 q}$
- Equating this energy to the initial energy of the α- particle we get
- E_α = $\frac{1}{2}mv^2 = \frac{1}{2}mv_m^2$ + $\frac{ZZ'e^2}{4\pi\varepsilon_0 q}$----------------------(1)
- or $\frac{v_m^2}{v^2}$ = 1- $\frac{ZZ'e^2}{2\pi\varepsilon_0 Mv^2 q}$ ---------------------------(2)

Scattering Cross Section

- Linear momentum of the α- particle at a great distance is Mv along the line QOR and hence the initial angular momentum is
- Mv. FG = Mv.b

- At A', the linear momentum Mv_m of the α particle is perpendicular to the axis FF'of the hyperbola so that the angular momentum at this point is
- Mv_m. FA' = Mv_mq

- Since the force acting on the $\alpha -$ particle is central its angular momentum must be conserved, thus

- Mv.b =Mv_mq-------------------(3)

Scattering Cross Section

- For the hyperbola we have
- OF = OF'= $\in$ OA' Where $\in > 1$ is the eccentricity
- Thus q= FA' = OF+OA' =OF$\left(1+\frac{1}{\in}\right)$
- For Conics, $\frac{1}{\in} = \cos\boldsymbol{\phi}$
- So that q = OF (1+ cos $\boldsymbol{\phi}$)
-
- Since sin$\boldsymbol{\phi}$ = FG/OF, we have
- q = FG (1 + cos $\boldsymbol{\phi}$)/ sin$\boldsymbol{\phi}$ = b (1 + cos $\boldsymbol{\phi}$)/ sin$\boldsymbol{\phi}$ --------(4)
- From Eqs, (3) and (4) we get
- $\frac{b}{q} = \frac{v_m}{v} = \frac{\sin\boldsymbol{\phi}}{1+\cos\boldsymbol{\phi}}$------------------------------ (5)

Scattering Cross Section

- From Equation (2) and (5) we have
- $1-\dfrac{ZZ'e^2}{2\pi\epsilon_0 Mv^2\, q} = \dfrac{\sin^2\phi}{(1+\cos\phi)^2} = \dfrac{1-\cos\phi}{1+\cos\phi}$
- Using the value of q from equation (5) we get
- $1-\dfrac{ZZ'e^2}{2\pi\epsilon_0 Mv^2\, b}\cdot\dfrac{\sin\phi}{(1+\cos\phi)} = \dfrac{1-\cos\phi}{1+\cos\phi}$
- Or, $\dfrac{ZZ'e^2}{2\pi\epsilon_0 Mv^2\, b}\sin\phi = 2\cos\phi$
- Finally b = $\dfrac{ZZ'e^2}{4\pi\epsilon_0 Mv^2}\tan\phi$ ---------------(6)
- Again since $\phi+\dfrac{\theta}{2}=\dfrac{\pi}{2}$, So, b = $\dfrac{ZZ'e^2}{4\pi\epsilon_0 Mv^2}\cot\dfrac{\theta}{2}$ -----------(7)
- From the expression of b it is clear that as b decreases scattering angle θ increases

Scattering Cross Section

- If we consider two cylindrical region having radii b and b+db in which the scattered $\alpha -$ particles paths are lying then the number of $\alpha -$ particle in this annular region in a given interval of time can be given as
- dN = N 2 π b db
- Or,
- $dN = -\pi N\left(\frac{ZZ'e^2}{4\pi\epsilon_0 Mv^2}\right)^2 cosec^3\frac{\theta}{2}\cos\frac{\theta}{2}\, d\theta$ --------(8)
- All these $\alpha -$ particles are scattered into the solid angle
- $d\boldsymbol{\Omega} = 2\pi \sin\theta\, d\theta = 4\pi\sin\frac{\theta}{2}\cos\frac{\theta}{2}\, d\theta$
- Thus the number scattered per unit solid angle is (for Z'=2)
- $\frac{dN}{d\boldsymbol{\Omega}} = N\left(\frac{Ze^2}{4\pi\epsilon_0 Mv^2}\right)^2 cosec^4\frac{\theta}{2}$ ------------------(9)
- Negative sign is dropped indicating reverse relation between b and θ.

Scattering Cross Section

- If Single $\alpha -$ particle scattered from unit area, it gives the probability of scattering per unit solid angle which can be given as
- $\frac{d\boldsymbol{\sigma}}{d\boldsymbol{\Omega}} = \left(\frac{Ze^2}{4\pi\epsilon_0 Mv^2}\right)^2 cosec^4 \frac{\theta}{2}$ --------------------------------(10)
- Above equation is known as Rutherford Scattering Formula.
- $\frac{Ze^2}{4\pi\epsilon_0 Mv^2}$ has the dimension of length its square has the dimension of area. Thus the equation (10) designated as the cross section of scattering per unit solid angle of differential scattering cross section.
- Integrating d$\boldsymbol{\sigma}$ over all the solid angle we can get the total scattering cross section

Number of Scattered $\alpha -$ particle on the unit area

- Number of Scattered $\alpha -$ particle on the unit area falling on the unit area of scatting foil is given as
-
- $N_s = \frac{\Delta N}{\Delta S} = \frac{Nnt}{r^2}\left(\frac{Ze^2}{4\pi\epsilon_0 Mv^2}\right)^2 cosec^4 \frac{\theta}{2}$
- Where N is the number of $\alpha -$ particle incident on the foil
- n is the number of scattering nuclei per unit volume of the foil.
- t is the thickness of the foil and r is the radius of sphere to obtain the solid angle.

Experimental verification of the Rutherford formula

- Geiger and Marsden verify the Rutherford formula of scattering experimentally and made the following conclusion
- $\boldsymbol{N_s} \propto \frac{1}{sin^4 \frac{\boldsymbol{\theta}}{2}}$
- $\boldsymbol{N_s} \propto$ t
- $\boldsymbol{N_s} \propto \frac{1}{E^2}$ (where E= $\frac{1}{\mathbf{2}}$ m$v^{\mathbf{2}}$ is the kinetic energy of the $\alpha -$ particle)
- $\boldsymbol{N_s} \propto Z^{\mathbf{2}}$

Discovery of the Neutron

- Rutherford proposed the atomic structure with the massive nucleus in 1911.
- which particles compose the nucleus was known only in 1932
- **Three** reasons why electrons cannot exist within the nucleus:

1) Nuclear size

The uncertainty principle puts a lower limit on its kinetic energy that is much larger that any kinetic energy observed for an electron emitted from nuclei (its actually the result of β-decay).

2) Nuclear spin

If a deuteron nucleus were to consist of protons and electrons, the deuteron must contain 2 protons and 1 electron. A nucleus composed of 3 fermions must result in a half-integral spin. But it has been measured to be 1. So no electrons can possible in the nucleus (but they apparently come out of certain nuclei)

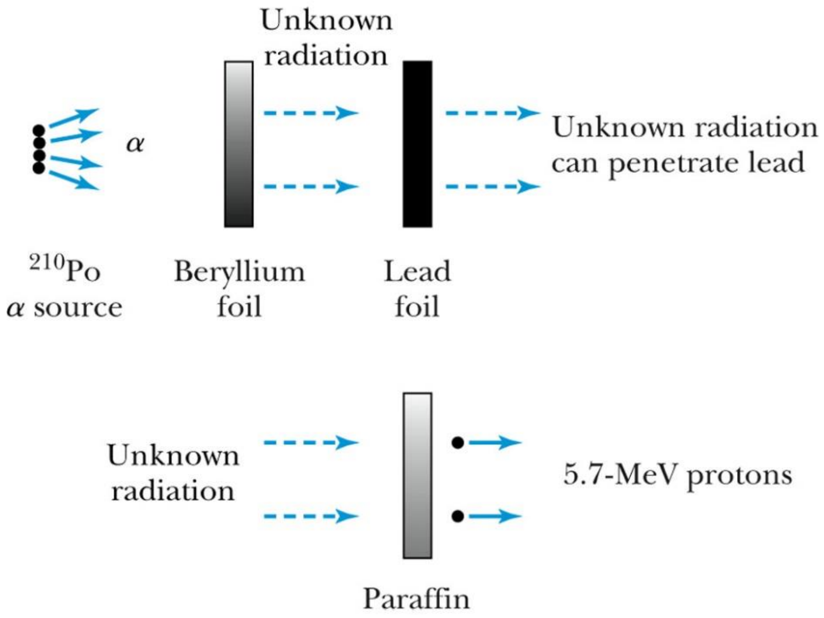
Discovery of the Neutron

3) Nuclear magnetic moment:

The magnetic moment of an electron is over 1000 times larger than that of a proton.

The measured nuclear magnetic moments are on the same order of magnitude as the proton's, so an electron canot be a part of the nucleus.

In 1930 the German physicists Bothe and Becker used a radioactive polonium source that emitted α particles. When these α particles bombarded beryllium, the radiation penetrated several centimeters of lead but was readily absorbed by paraffine wax



Discovery of the Neutron

- In 1932 Chadwick proposed that the new radiation produced by α + Be consisted of neutrons. His experimental data estimated the neutron's mass as somewhere between 1.005 u and 1.008 u, not far from the modern value of 1.0087 u.

- The electromagnetic radiation (photons) are called **gamma rays** which have energies on the order of MeV.

- Curie and Joliot performed several measurements to study penetrating high-energy gamma rays.

- There are also electrons (and positrons) emerging from atoms, beta rays (but they are not constituents of the nucleus themselves)