Environmental Engineering- I (BCE-26) Online Lecture

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The population of a town as per the Census records are given below for the years 1921 to 1981. Assuming that the scheme of water supply will comment to function from 1986, it is required to estimate the population 30 years hence, i.e. in 2016 and also the intermediate population 15 years after 1986, i.e. 2001.

Year	Population	Increment
1921	40,185	
1931	44,522	4337
1941	60,395	15,873
1951	75,614	15219
1961	98,886	23,272
1971	124,230	25,344
1981	158,800	34,570
	Total	**118,615**
	Average	**19,615**

1. Arithmetical Progression Method

Increase in population from 1921 to 1981 i.e. in 6 decades = 1,58,800
- 40,185
1,18615

or increase per decade = 1/6 x 118,615
= 19,769

Population in 2001 = Population in 1981 + Increase for 2 decades
= 1588,00 + 2×19,769
=158,800 +39,538
= 198,338

Population in 2016 = Population in 1981 + Increase for 3.5 decades
= 158,800 + 3.5 × 19,769
= 227,992

2. Geometrical Progression Method

Rate of growth per decade between 1931 and 1921 = 4337/40,185 = .108

1941 and 1931	= 15,873/44,522	= 0.356
1951 and 1941	= 15,219/60,395	= 0.252
1961 and 1951	= 23,272/75,614	= 0.308
1971 and 1961	= 25,344/98,886	= 0.256
1981 and 1971	= 34,570/1,24,230	= 0.278

Geometric mean, $r_g = \sqrt[6]{0.108 \times 0.356 \times 0.252 \times 0{,}308 \times 0.256 \times 0.278}$

Assuming that the future growth follows the geometric mean for the period 1921 to 1981 = 0.2442

Population in 2001 = Population in 1981 × $(1 + r_g)^2$

158,800× $(1 .2442)^2$

2,45,800

Population in 2016 = Population in 1981 $(1 + r_g)^{3.5}$

158,800× $(1.2442)^{3.5}$ = 3,05,700

Year	Population	Increase		Incremental Increase Y
1921	40,185			
1931	44,522	4337		
1941	60,395	15,873	+	11,536
1951	75,614	15,219	-	654
1961	98 886	23,272	+	8,053
1971	124,230	25,272	+	2072
1981	1,58,800	34,570	+	9226
	Total	**118,615**		**30,233**
	Average	= 1/6×118,615 = 19,769		= 1/5×30,233 = 6,047

$$P_n = P + nx + \frac{n(n+1)Y}{2}$$

$$P2001 = P1981 + 2\times19769 + \frac{2\times3\times6047}{2}$$

= 1,58,800 +39,538+18141

= 2,16,479

P2016 =P1981+3.5×19769+24188

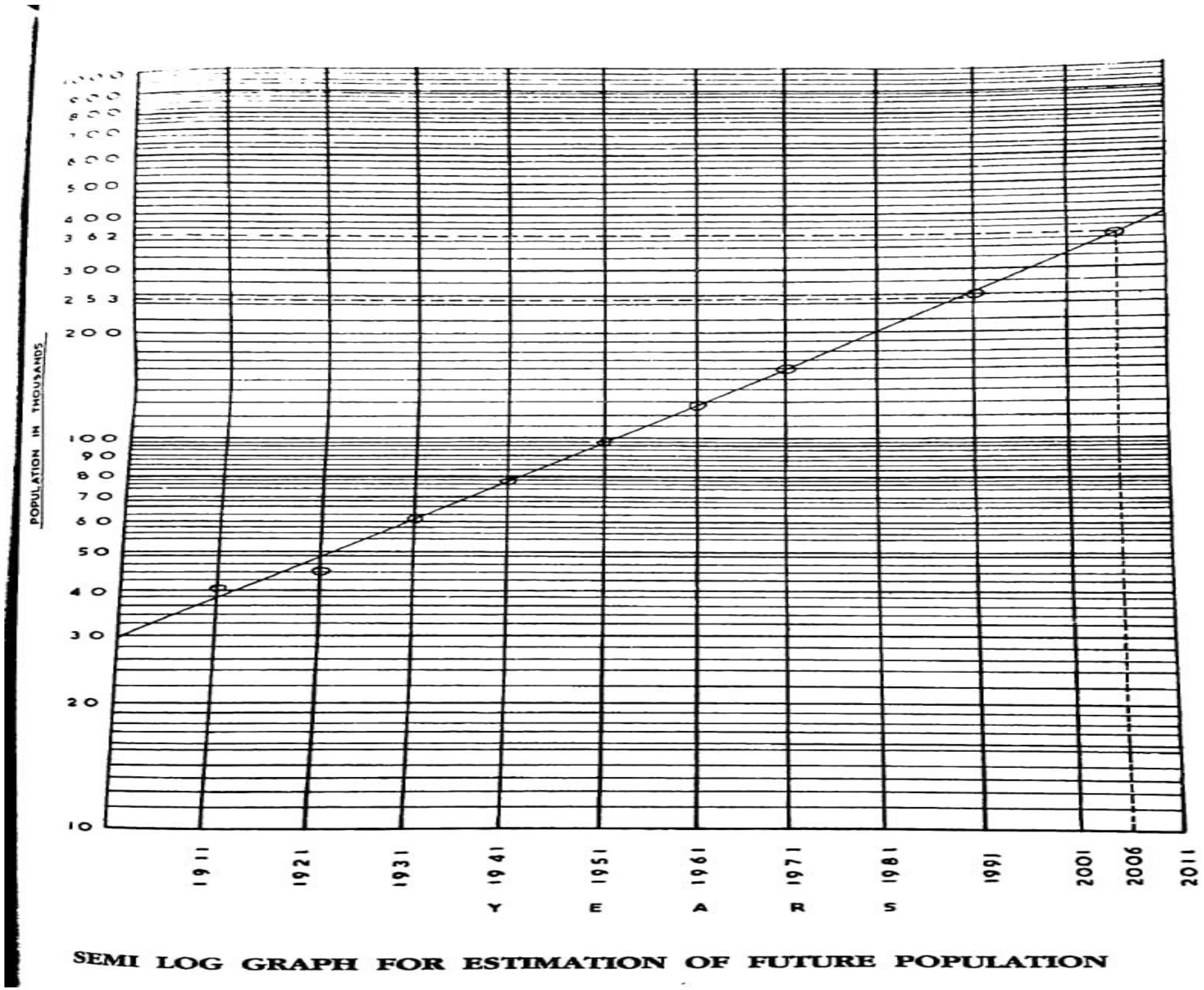
= 1,58,800 +69,192+24188

=2,52180

Graphical Projection Method From the graph presented on the following page, the figures for 2001 and 2016 obtained are as follows:

2001- 253000

2016- 362000



SEMI LOG GRAPH FOR ESTIMATION OF FUTURE POPULATION

Example. Work out the population of the year 2,000 from the data given below, using simple graphical method.

Table

Year	Population
1880	25,000
1890	27,500
1900	33,000
1910	39,000
1920	45,000
1930	54,500
1940	61,000

Solution. The graph between time and population is plotted from given in table as shown in Fig.

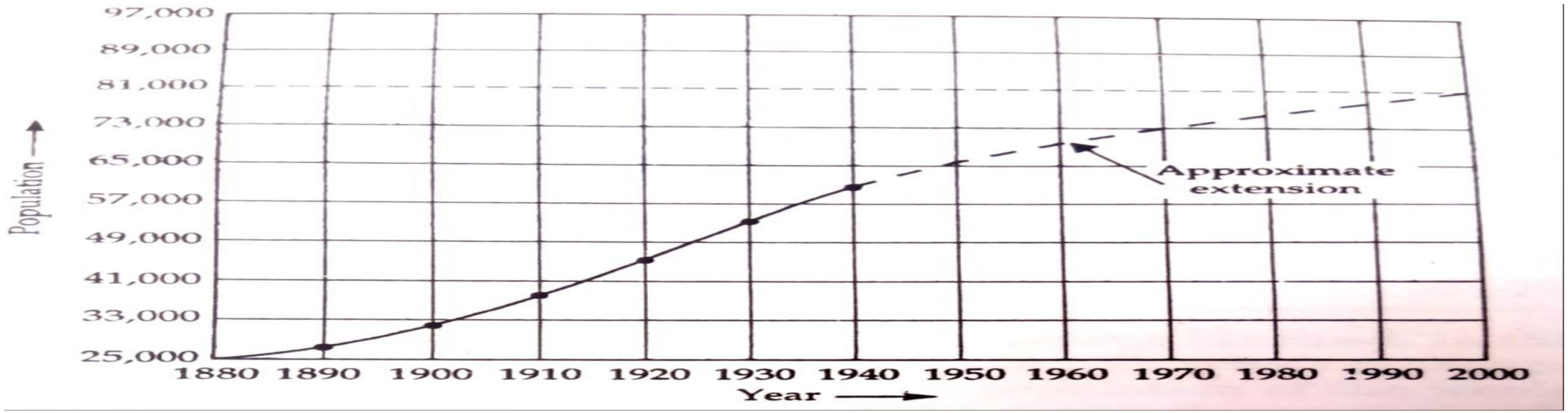


Fig. Graph between time and population for example

2.10.9. The Logistic Curve Method. It was explained earlier that under normal conditions, the population of a city shall grow as per the *logistic curve*, shown in Fig. 2.3. P.F. Verhulst has put forward a mathematical solution for this logistic curve. According to him, the entire curve *AD* (Fig. 2.3) can be represented by an autocatalytic first order equation, given as :

$$\log_e\left(\frac{P_s - P}{P}\right) - \log_e\left(\frac{P_s - P_0}{P_0}\right) = -K\,P_s\,.\,t \qquad \text{...(2.20)}$$

where P_0 = The population at the start point of the curve *A*

P_s = Saturation population

P = Population at any time *t* from the origin *A*

K = Constant.

From Eq. (2.20), we get

$$\log_e\left[\left(\frac{P_s - P}{P}\right)\times\left(\frac{P_0}{P_s - P_0}\right)\right] = -K\,P_s.t$$

or
$$\left(\frac{P_s - P}{P}\right)\left(\frac{P_0}{P_s - P_0}\right) = \log_e^{-1}(-K.P_s.t)$$

or
$$\frac{P_s - P}{P} = \left[\frac{P_s - P_0}{P_0}\right]\log_e^{-1}(-K.P_s.t)$$

or
$$\frac{P_s}{P} - 1 = \left[\frac{P_s - P_0}{P_0}\right]\log_e^{-1}(-K.P_s.t)$$

or
$$\frac{P_s}{P} = 1 + \left[\frac{P_s - P_0}{P_0}\right]\log_e^{-1}(-K.P_s.t)$$

or
$$P = \frac{P_s}{1 + \frac{P_s - P_0}{P_0}\log_e^{-1}(-KP_s.t.)} \qquad \text{...(2.21)}$$

Substituting $\frac{P_s - P_0}{P_0} = m$ (a constant)

and $-K.P_s = n$ (another constant)

we get

$$P = \frac{P_s}{1 + m\,\log_e^{-1}(nt)} \qquad \text{...(2.22)}$$

This is the required equation of the logistic curve. McLean further suggested that if only three pairs of characteristic values P_0, P_1, P_2 at times $t = t_0 = 0$, t_1, and $t_2 = 2t_1$ extending over the useful range of the census populations, are chosen, the saturation value P_s and the constants *m* and *n* can be evaluated from three simultaneous equations, as follows :

$$P_s = \frac{2P_0P_1P_2 - P_1^2(P_0 + P_2)}{P_0P_2 - P_1^2} \qquad \text{...(2.23)}$$

$$m = \frac{P_s - P_0}{P_0} \qquad \text{...(2.24)}$$

$$n = \left(\frac{1}{t_1}\right)\log_e\left[\frac{P_0(P_s - P_1)}{P_1(P_s - P_0)}\right] \qquad \text{...(2.25)}$$

$$= \frac{2.3}{t_1}\log_{10}\left[\frac{P_0(P_s - P_1)}{P_1(P_s - P_0)}\right] \qquad \text{...(2.25a)}$$

Knowing P_0, P_1 and P_2 from census data and using them in these equations, the values of P_s, *m* and *n* are known, and the equation of the logistic curve (Eq. 2.22) is thus known. From that, the population *P* at any time *t* can then be obtained, as explained in the example given below.

Example 2.12. *In two periods of each of 20 years, a city has grown from 30,000 to 1,70,000 and then to 3,00,000. Determine, (a) the saturation population ; (b) the equation of the logistic curve ; (c) the expected population after the next 20 years.*

Solution. In this question, we have

P_0 = 30,000 $\qquad t = 0$

P_1 = 1,70,000 $\qquad t_1$ = 20 yrs.

P_2 = 3,00,000 $\qquad t_2$ = 40 yrs.

Using Eq. (2.23), we have

$$P_s = \frac{2P_0P_1P_2 - P_1^2(P_0 + P_2)}{P_0P_2 - P_1^2}$$

$$\therefore \quad P_s = \frac{2\times 30{,}000\times 1{,}70{,}000\times 3{,}00{,}000 - (1{,}70{,}000)^2[30{,}000 + 3{,}00{,}000]}{30{,}000\times 3{,}00{,}000 - (1{,}70{,}000)^2}$$

$$= \frac{[2\times 30\times 170\times 300 - (170)^2(330)]\,10^9}{[30\times 300 - (170)^2]\,10^6}$$

$$= \frac{170\,[(18{,}000) - 170\times 330]\,10^3}{9{,}000 - 28{,}900}$$

$$= \frac{(-)\,170\times 38{,}100}{(-)\,19{,}900} = 3{,}26{,}000$$

(*a*) Hence, the saturation population = 3,26,000. **Ans.**

Using Eq. (2.24), we get

$$m = \frac{P_s - P_0}{P_0}$$

$$= \frac{3,26,000 - 30,000}{30,000} = \frac{2,96,000}{30,000} = \mathbf{9.87}$$

Using Eq. (2.25*a*), we get

$$n = \frac{2.3}{t_1}\log_{10}\left[\frac{P_0(P_s - P_1)}{P_1(P_s - P_0)}\right]$$

$$= \frac{2.3}{20}\log_{10}\left[\frac{30,000\,(3,26,000 - 1,70,000)}{1,70,000\,(3,26,000 - 30,000)}\right]$$

$$= \frac{2.3}{20}\log_{10}\left(\frac{30 \times 156}{170 \times 296}\right) = \frac{2.3}{20}\log_{10} 0.093$$

$$= \frac{2.3}{20} \times (\bar{2}.968) = \frac{2.3}{20}(-1.032)$$

$$= -\mathbf{0.119.}$$

(*b*) Substituting the values of P_s, m and n in Eq. (2.22), we get the required equation of logistic curve, as

$$\left[P = \frac{3,26,000}{1 + 9.87\log_e^{-1}(-0.119\,t)}\right] \quad \textbf{Ans.} \qquad \ldots(2.26)$$

(*c*) When $t = 60$ yrs., what is P ?

Substituting $t = 60$ in Eq. (2.26), we get

$$P = \frac{3,26,000}{1 + 9.87\log_e^{-1}(-0.119 \times 60)}$$

or

$$P = \frac{3,26,000}{1 + 9.87\log_e^{-1}(-7.14)} = \left[\frac{3,26,000}{1 + 9.87\,x}\right]$$

where, $x = \log_e^{-1}(-7.14)$.

Now, to evaluate $\log_e^{-1}(-7.14)$, we proceed as follows :

$$\log_e^{-1}(-7.14) = x$$

or $$\log_e^{x} = -7.14$$

or $$2.3\log_{10}^{x} = -7.14$$

or $$\log_{10}^{x} = \frac{-7.14}{2.3} = -3.1 = \bar{4}.9$$

Taking antilog, we get

$$x = 0.000795$$

$$\therefore \quad P = \frac{3,26,000}{1 + 9.87 \times 0.000795}$$

$$= \frac{3,26,000}{1 + 0.000776} = \frac{3,26,000}{1.00776} = \mathbf{3,23,000}$$

Hence, the population after 20 more years will be = **3,23,000. Ans.**

TYPES OF INTAKE

Simple Submerged Intakes

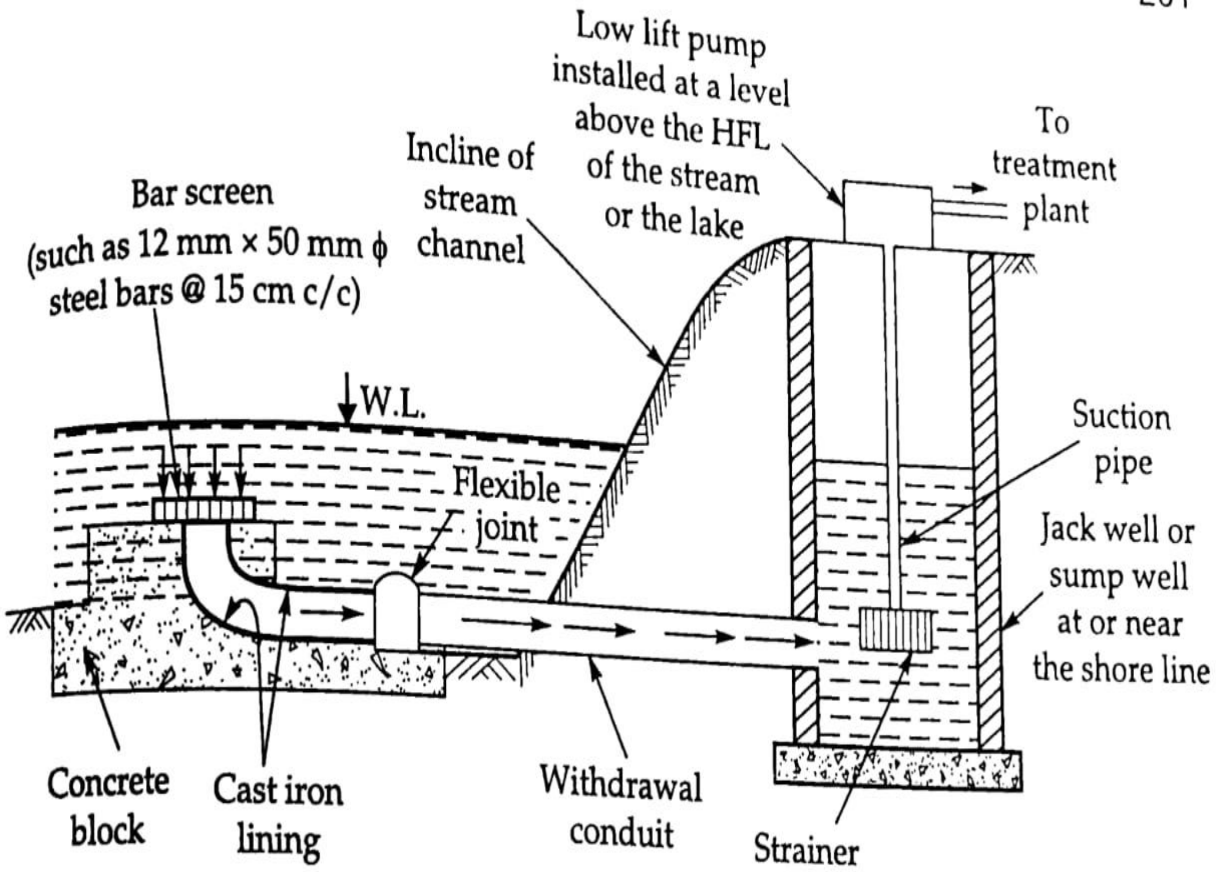


Fig. 5.1. Simple concrete block—submerged intake.

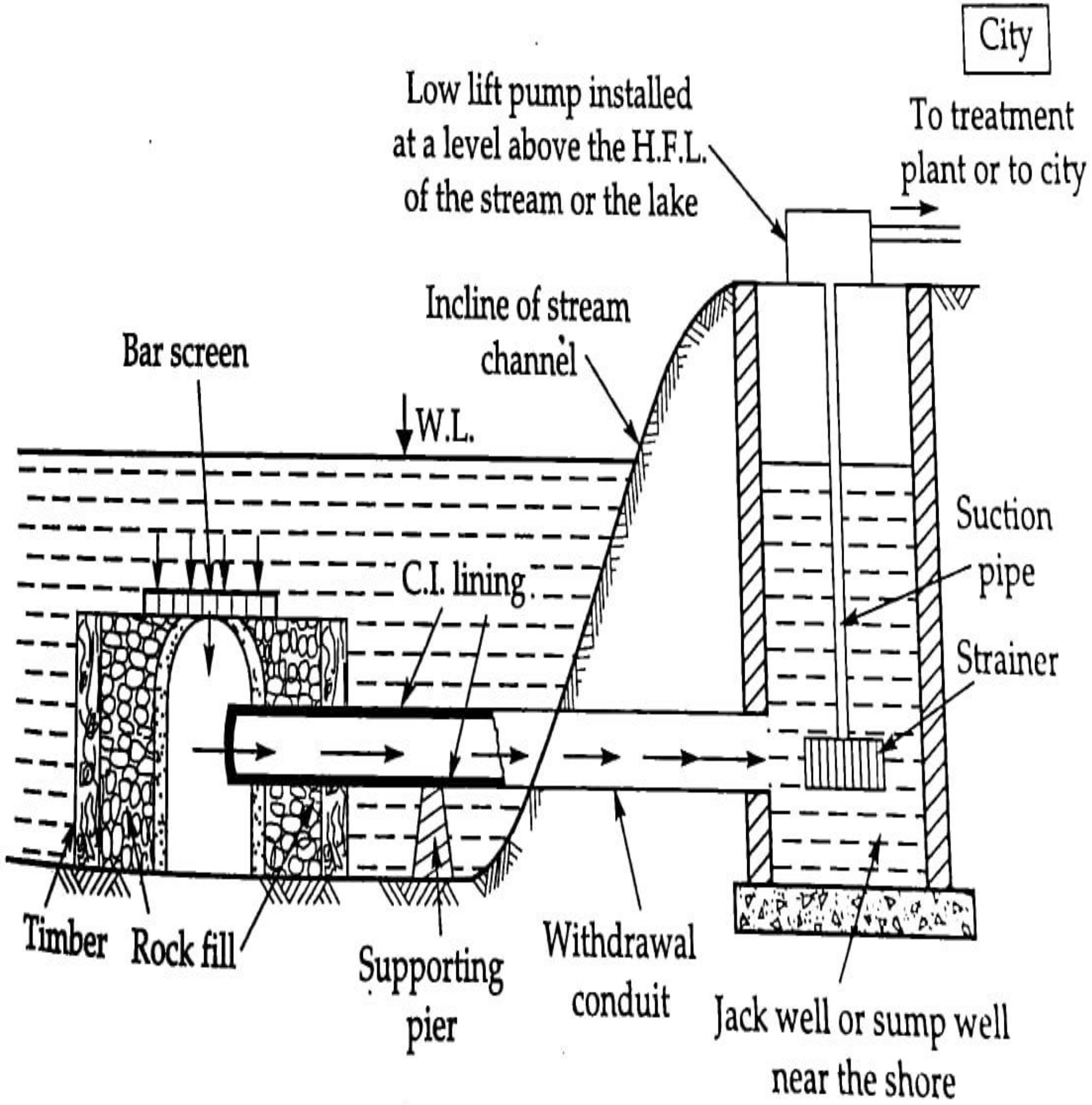


Fig. 5.2. Rock filled timber crib—submerged intake.

INTAKE TOWERS

1. Wet Intake Tower

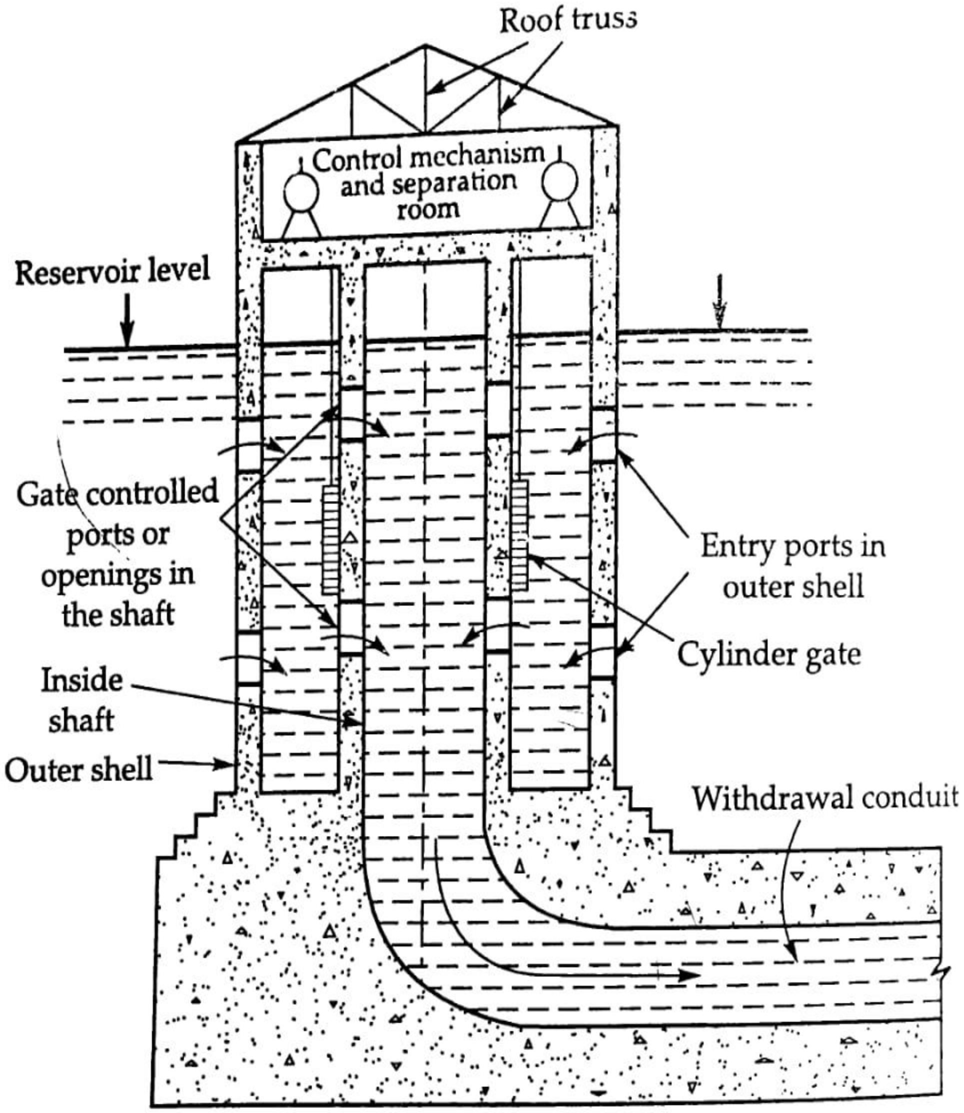


Fig. 5.3. Wet intake tower standing in the river or reservoir.

2. Dry Intake Tower

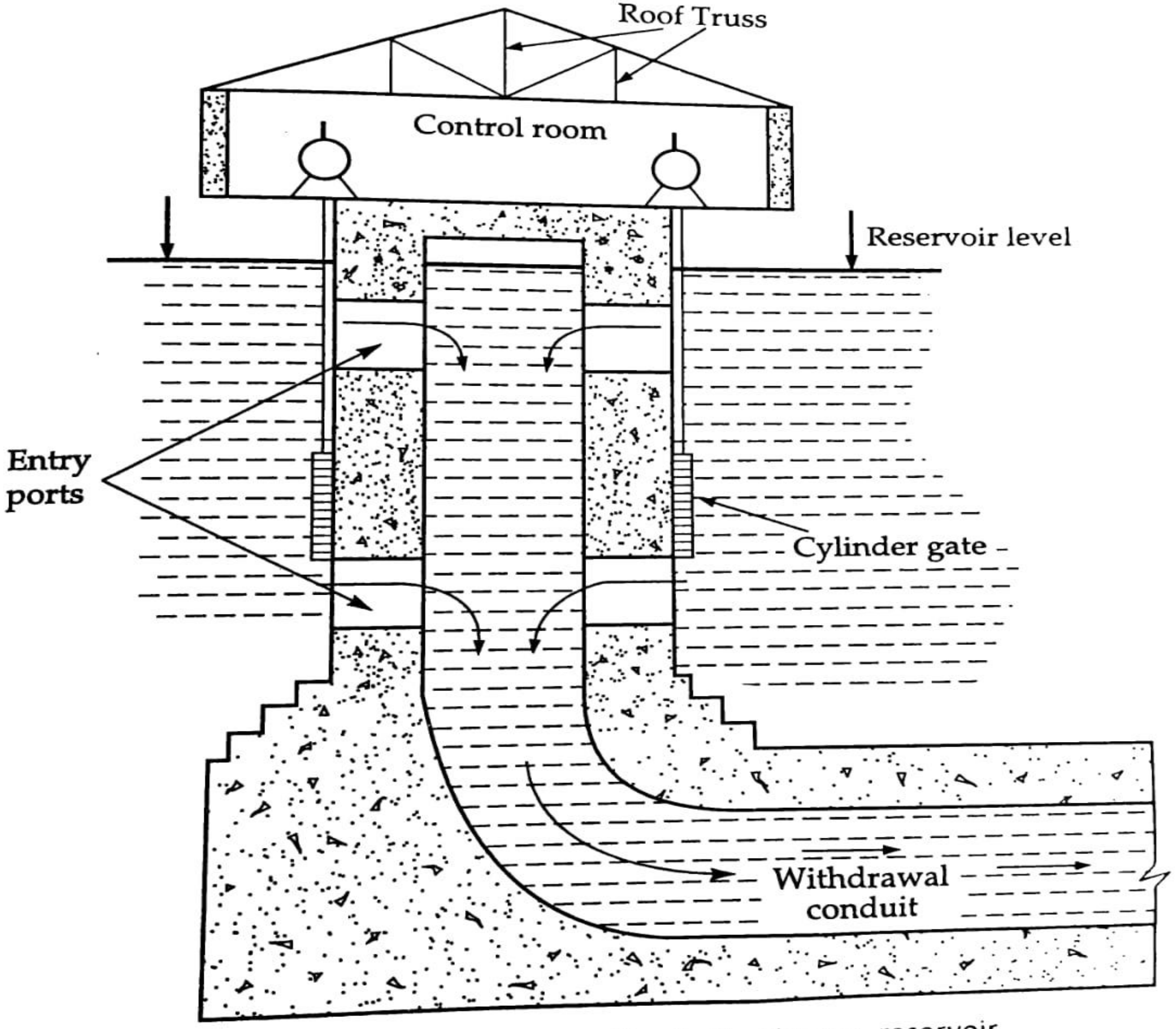


Fig. 5.4. Dry intake tower standing in the river or reservoir.

Medium Sized River Intake Structures

Medium sized river intake structures are generally constructed to withdrawing water from almost all rivers, and are a via media between the submerged intake (usually adopted for small streams) and the intake towers (usually adopted for reservoirs).

While the canal intakes are discussed in the next article River intake structure can be broadly classified into the following two types.

I. Twin well type of intake structure.

II. Single well type of intake structure.

Typical Twin Well Type of River Intake Structure

This is the most commonly used type of river intake, which is generally constructed in almost all types of rivers, where the river water hugs the river bank.

Such condition in usually available on non-alluvial rivers, while in meandering alluvial rivers, such a condition can be obtained by constructing a weir across the river as to store water up to pond level, thus making it available near the river bank on the under-sluice side.

Typical details of such an intake structure are shown in Fig

Typical river intake structure consists of:

I. an inlet well

II. an inlet pipes

III. a jack well

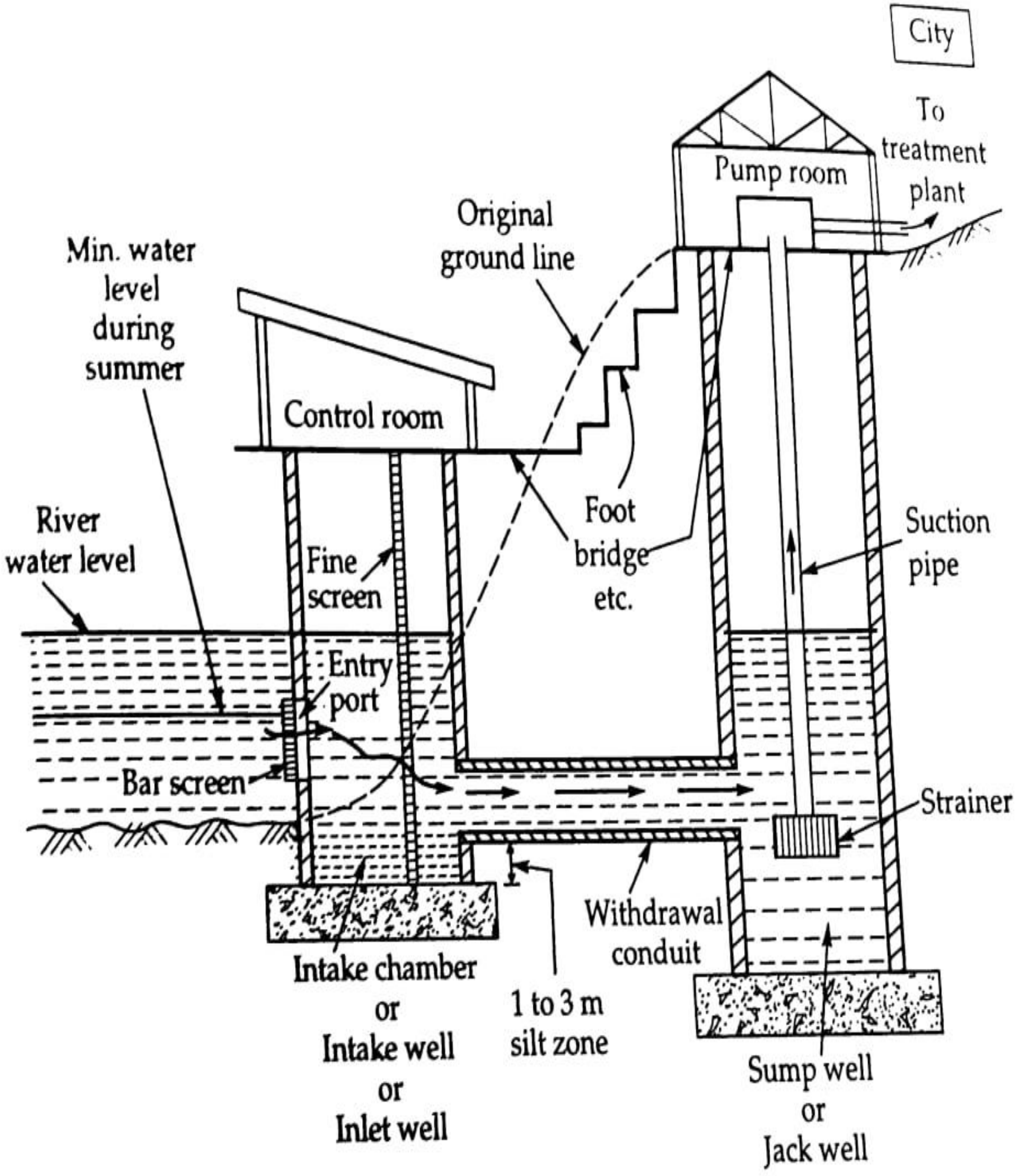


Fig. Section of a typical twin well type of a river intake

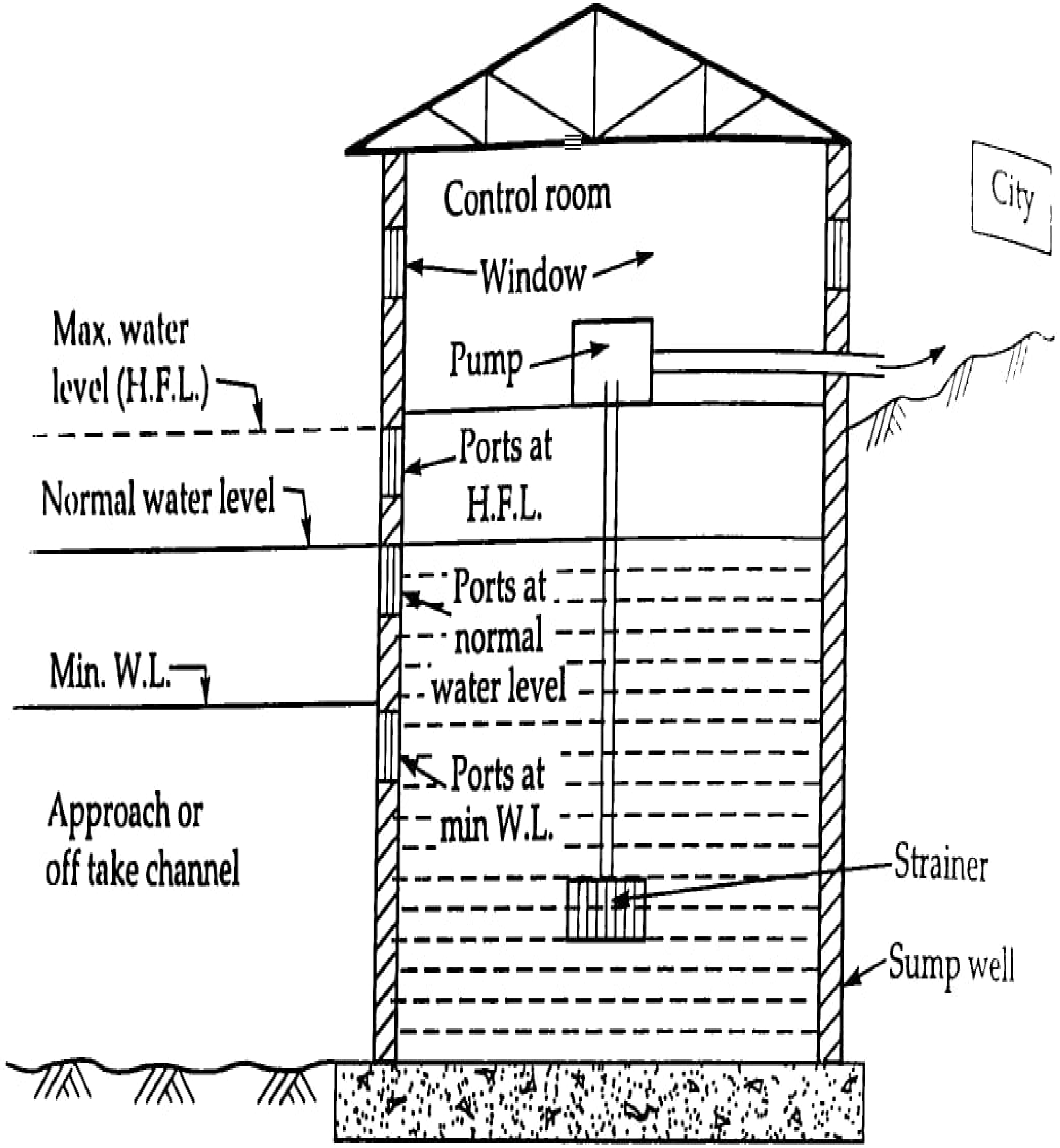


Fig. Single well type of a river intake structure

Canal Intake

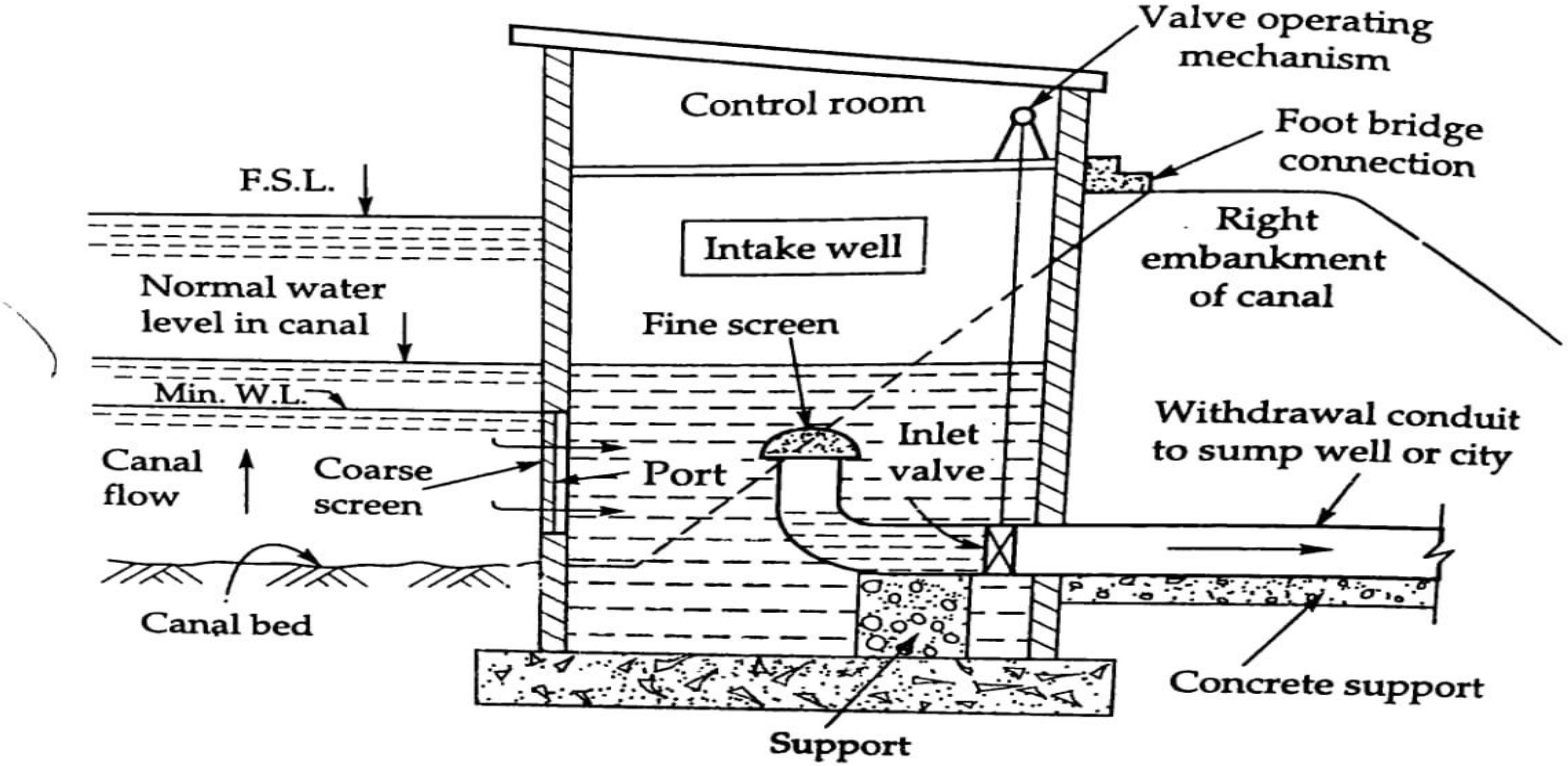


Fig. Canal Intake well

Intake for Sluice-ways of Dams

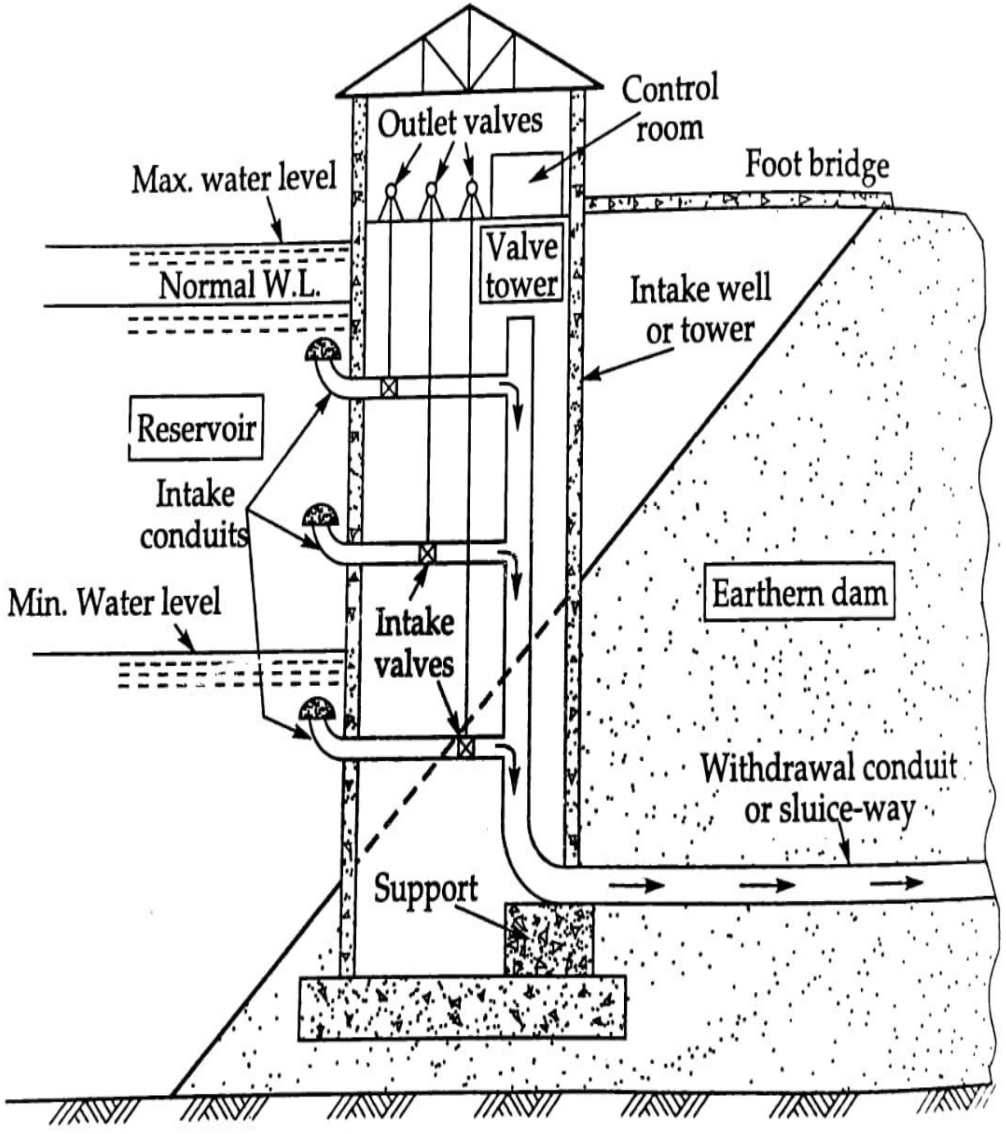


Fig. Valve tower situated at the upstream toe of earthen dam

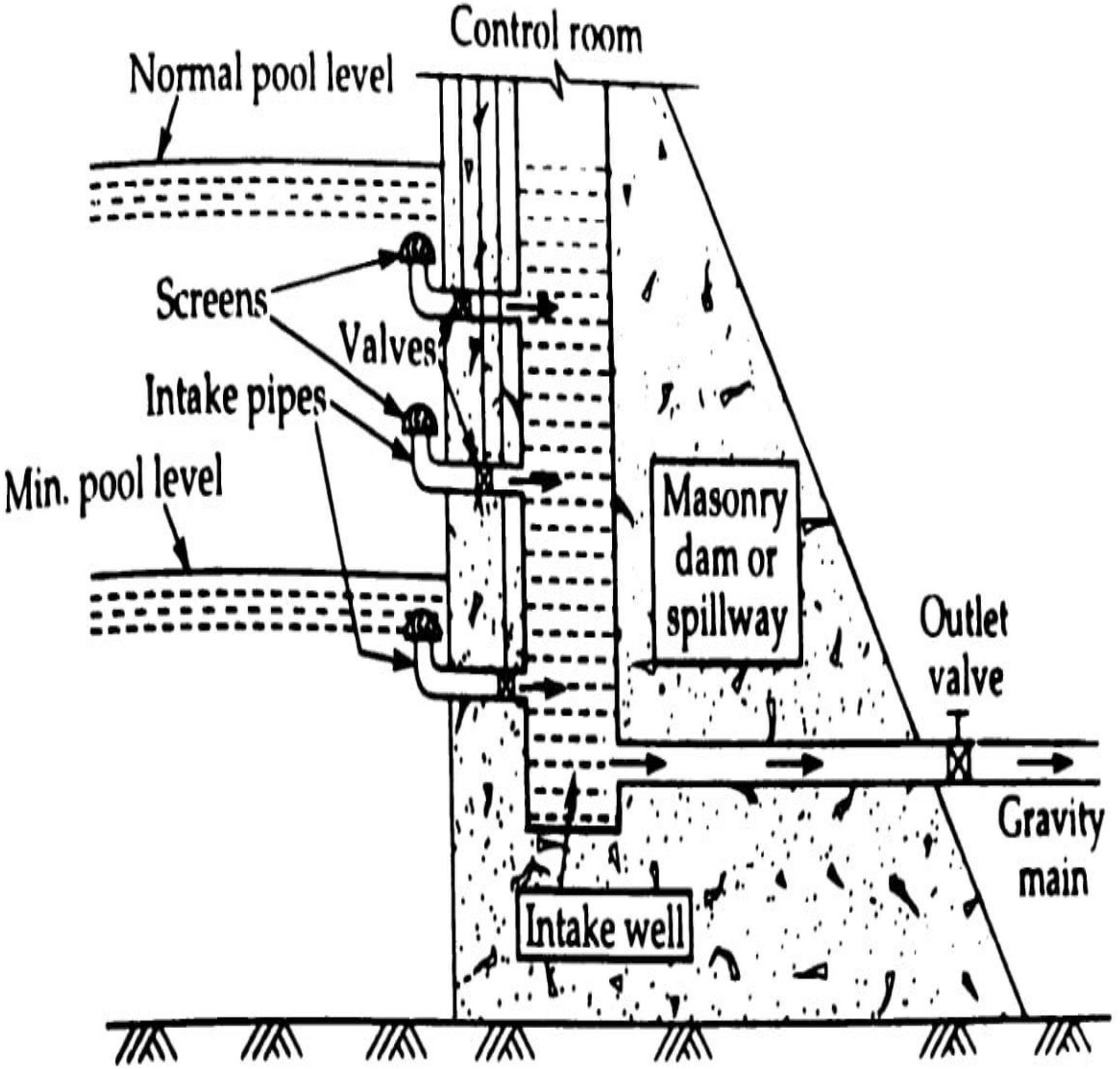


Fig. intake well or valve tower for concrete gravity or masonry dams

Intake for Sluice-ways of Dams

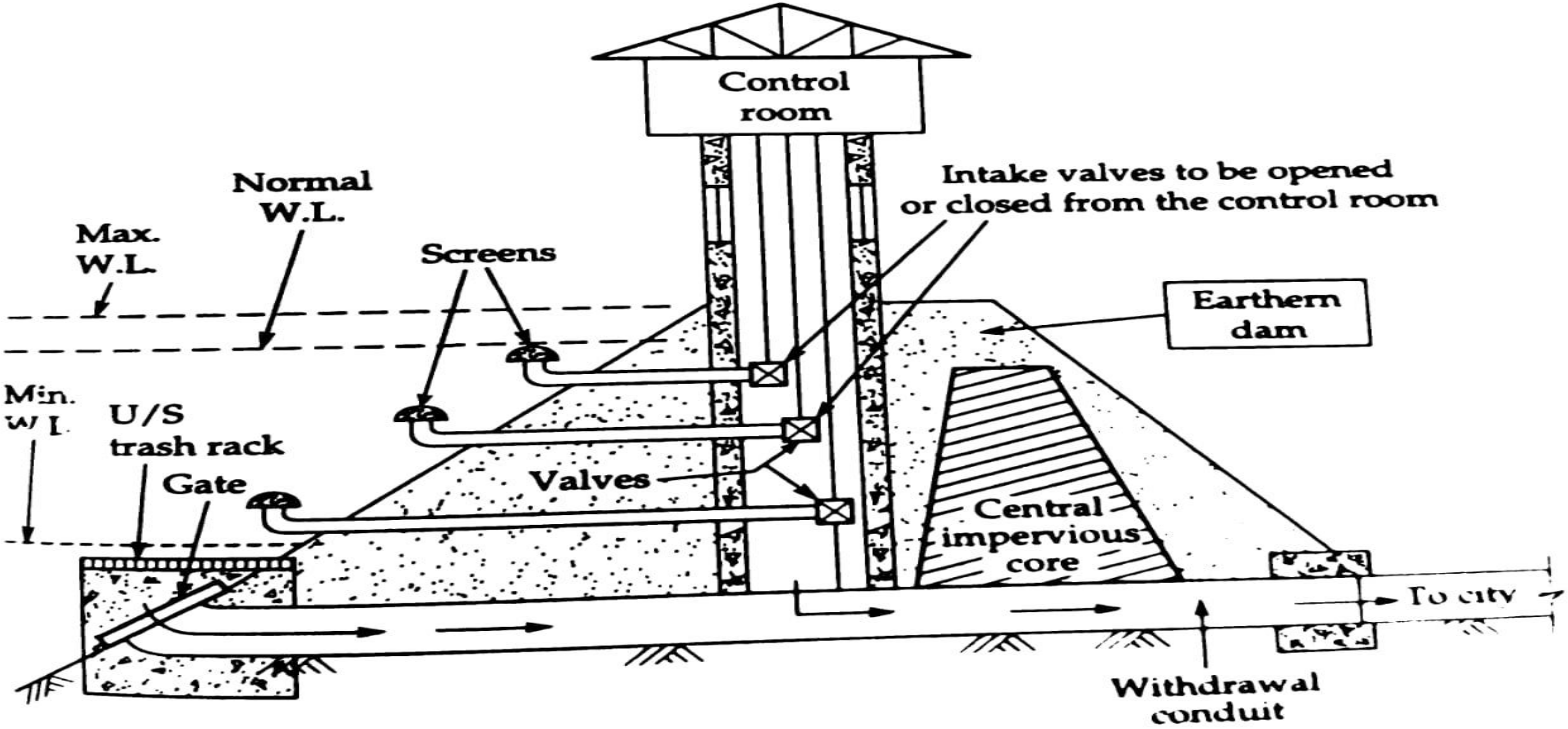


Fig. Valve Tower situated within an earthen dam