Log-distance path loss model

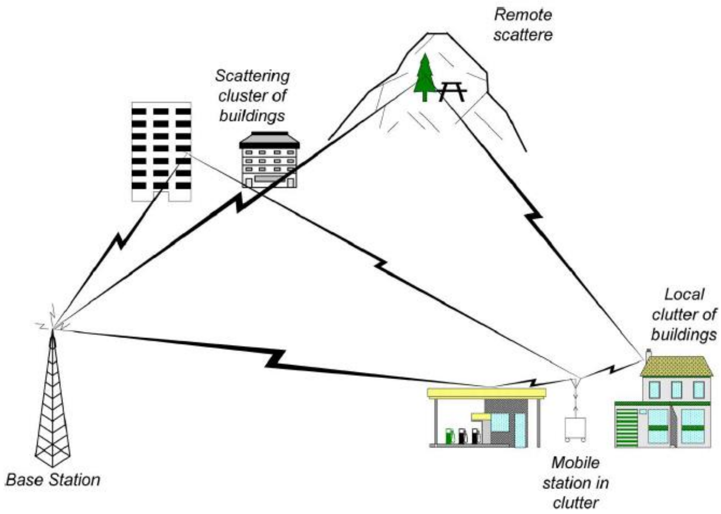

Figure 5

Let P_0 be the reference power at the distance d_0 from the Tx and n is the path loss exponent.

$$\begin{aligned} P_r(d) &\alpha (1/d)^n \\ P_r(d) &= k \times (1/d)^n \\ P_0(d_0) &= k \times (1/d_0)^n \end{aligned} \tag{6.1}$$

From (6.1), taking the ratio of P_r and P_0, we have

$$P_r(d) = P_0 \left(\frac{d_0}{d} \right)^n \tag{7}$$

Let $P_t = \alpha \times P_0$ and hence $P_0 = P_t / \alpha$. Thus, received field is given as

$$P_r(d) = (P_t / \alpha) \left(\frac{d_0}{d} \right)^n \Rightarrow \frac{P_r}{P_t} = \left(\frac{1}{\alpha} \right) \left(\frac{d_0}{d} \right)^n$$

And hence path loss is given as

$\frac{P_t}{P_r} = \alpha \left(\frac{d}{d_0} \right)^n$ or in dB, Path loss is given as

$$\overline{P}_{loss,dB}(d) = \overline{P}_{loss,dB}(d_0) + 10 n \log(d / d_0) \tag{8}$$

It may be noted that the first term $\overline{P}_{loss,dB}(d_0)$ (which is equal to 10 log (P_t/P_r)) can simply be obtained by considering free-space model.

Here, it is worth mentioning the difference between path loss, shadowing effect (also called large-scale fading) and small-scale fading. Consider the following general expression of received power given as

$$\frac{p_t}{p_r} = \left[\left\{ k\left(d / d_0\right)^n \right\} \times s \times m \right] \tag{9}$$

The term in curly bracket is called path loss term which depends only upon distance between Tx and Rx and path loss exponent n. For different scenario such as rural area, sub-urban region and urban region, the path loss exponent varies. Thus path loss plot may get a discontinuity if mobile moves from one geographical region to other (or one cell to another cell) for which n is different. It may be quite possible that in the given region (such as urban or sub-urban), the height of the buildings may vary significantly. Thus, when the mobile moves, at a time, there may be direct LOS path and other time the LOS path may be blocked (or shadowed). Thus, the received field will fluctuate. In order to account for this effect, the deterministic path loss term in (9) is multiplied by a random variable. This second term 's' is called shadowing parameter which is a log-normal distributed random variable. This fluctuation can vividly be observed in the range of several wavelength. At the local level, this total signal $\left\{k\left(d_0 / d\right)^n\right\} \times s$ is further getting fluctuated due to a multipath fading which is due to large number of multipath components being added constructively or destructively even with slight movement of mobile (in the range of λ/2). So if two multipath components are meeting with opposite phase, a destructive interference will occur. If they are meeting with same phase, then, constructive interference will occur. Hence, if we obtain spatial average of the term in (9) with respect to m and taking and normalize the average of m, we will get

$$S = E_m\left[\frac{p_t}{p_r}\right] = E_m\left[\left\{ k\left(d / d_0\right)^n \right\} \times s \times m \right] = \left[\left\{ k\left(d / d_0\right)^n \right\} \times s \right] \tag{10}$$

Similarly, if we obtain spatial average of the term in (10) with respect to s and taking and normalize the average of s, we will get

$$E_s[S] = PL = E_s\left[\left\{ k\left(d / d_0\right)^n \times s \right\} \right] = \left[\left\{ k\left(d / d_0\right)^n \right\} \right] \tag{11}$$

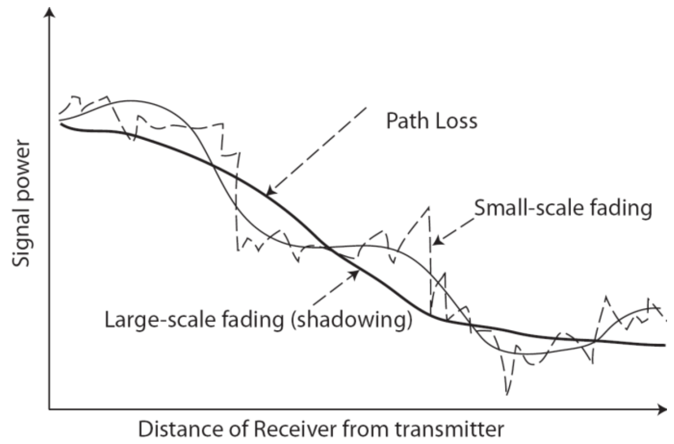

Figure 6: Path Loss vs Large-scale and small-scale fading

Log-normal Shadowing

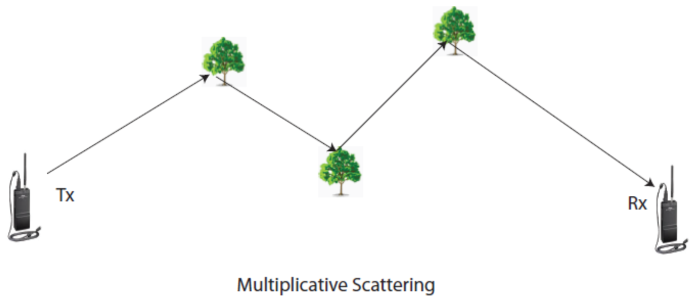

Fig. 1: Lognormal Shadowing

If X is a random variable having Gaussian distribution with mean m and variance σ_X^2 , then, a random variable Y defined as $Y=e^X$ will have a distribution called 'log-normal distribution' given as

$$p(y) = \frac{1}{\left(\sqrt{2\pi\sigma_X^2}\right)y} \exp\left[-(\ln y - m)^2 / 2\sigma_X^2\right] \qquad y \geq 0 \tag{12}$$

The basic idea of a shadowed channel being modeled as log-normal distribution is that the signal transmitted from the Tx arrives at the Rx after multiple diffractions and reflections. Let us assume that the transmitted signal E_0 goes through M number of reflections and N number of diffractions, then, the received signal can be expressed as

$$E_r = E_o \times \prod_{m=1}^{M} R_m \prod_{n=1}^{N} D_n \exp(-jkr_{total}) \Rightarrow |E_r| = E_o \times \prod_{m=1}^{M} R_m \prod_{n=1}^{N} D_n$$

If we take log on both the side of above equation, then, $\log|E_r| = \log(E_o) + \sum_{m=1}^{M} R_m + \sum_{n=1}^{N} D_n$. Now, assuming $M \to \infty, N \to \infty$, we can apply central-limit theorem and hence, the term $\log|E_r|$ will be normally distributed or in other word, $|E_r|$ will be log-normally distributed.

First and second moments of the LN random variable of (12) is given as

$$E[Y] = e^{m+\frac{\sigma_X^2}{2}}$$

$$\mathrm{var}[Y] = e^{2m+\sigma_X^2}(e^{\sigma_X^2} - 1)$$

The PDF in (12) can also be expressed in decibel form after slight mathematical manipulation and is given by

$$p(\gamma) = \frac{\xi}{\sigma_d \gamma \sqrt{2\pi}} e^{-\frac{(10\log_{10}\gamma - \mu_d)^2}{2(\sigma_d)^2}} \tag{13}$$

where $\mu_d = \xi\mu$ and $\sigma_d = \xi\sigma, \xi = 10/\ln(10) = 4.3429$

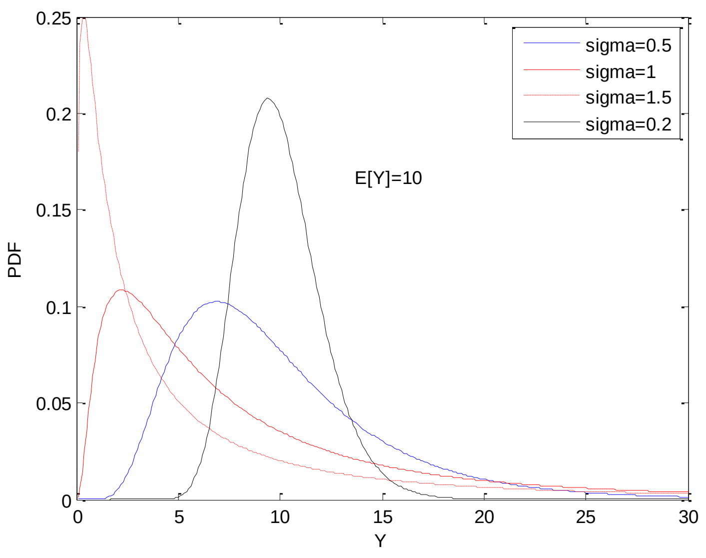

Fig. 3: Lognormal Shadowing

It may be noted that in Fig. 3, the plot of lognormal pdf has been obtained for several values of shadowing parameters. Here, the average power of signal is taken as 10. It is noted that as shadowing parameter decreases (channel condition is improving), the plots get shifted right side with decrease in the width of the plot indicating an improvement in the channel condition

The model in (8) gives the pathloss prediction at the receiver on an average sense (as is obvious from (9)-(11)). In a dense urban scenario, when a receiver is surrounded by innumerable scattering object and either Rx is moving or the small objects nearby are moving, then, the received signal at the Rx will be fluctuating in nature and should be modeled with statistical parameters as given in (9) where s is log-normal random variable (shadowing parameter) and m is RV due to multipath components (Small-scale fading). Taking log on both the side in (9) and noting that log (s) is a Gaussian random variable denoted as X with mean 0 dB and variance as σ^2, (in short: $X \sim N(0,\sigma^2)$), we have path loss model (in dB) at the receiver is given as