Control Systems

Subject Code: BEC-26 | Third Year ECE

Unit-III

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Lecture 2

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Example 2

For the given transfer function,

$$T.F. = \frac{(s+2)}{s(s+4)(s^2+6s+25)}$$

Find: (i) Poles (ii)Zeros

(iii) Pole-zero Plot (iv) Characteristics Equation

Solution: (i)Poles

The poles can be obtained by equating denominator with zero

$$s(s+4)(s^2+6s+25)=0$$

$$\therefore s=0$$

$$\therefore s+4=0 \qquad \therefore s=-4$$

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Example 2 Cont

$$s(s+4)(s^2+6s+25)=0$$

$$roots=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$\therefore s=-3+j4$$

$$\therefore s=-3-j4$$

The poles are s= 0, -4, -3+j4, -3-j4

(ii) Zeros:

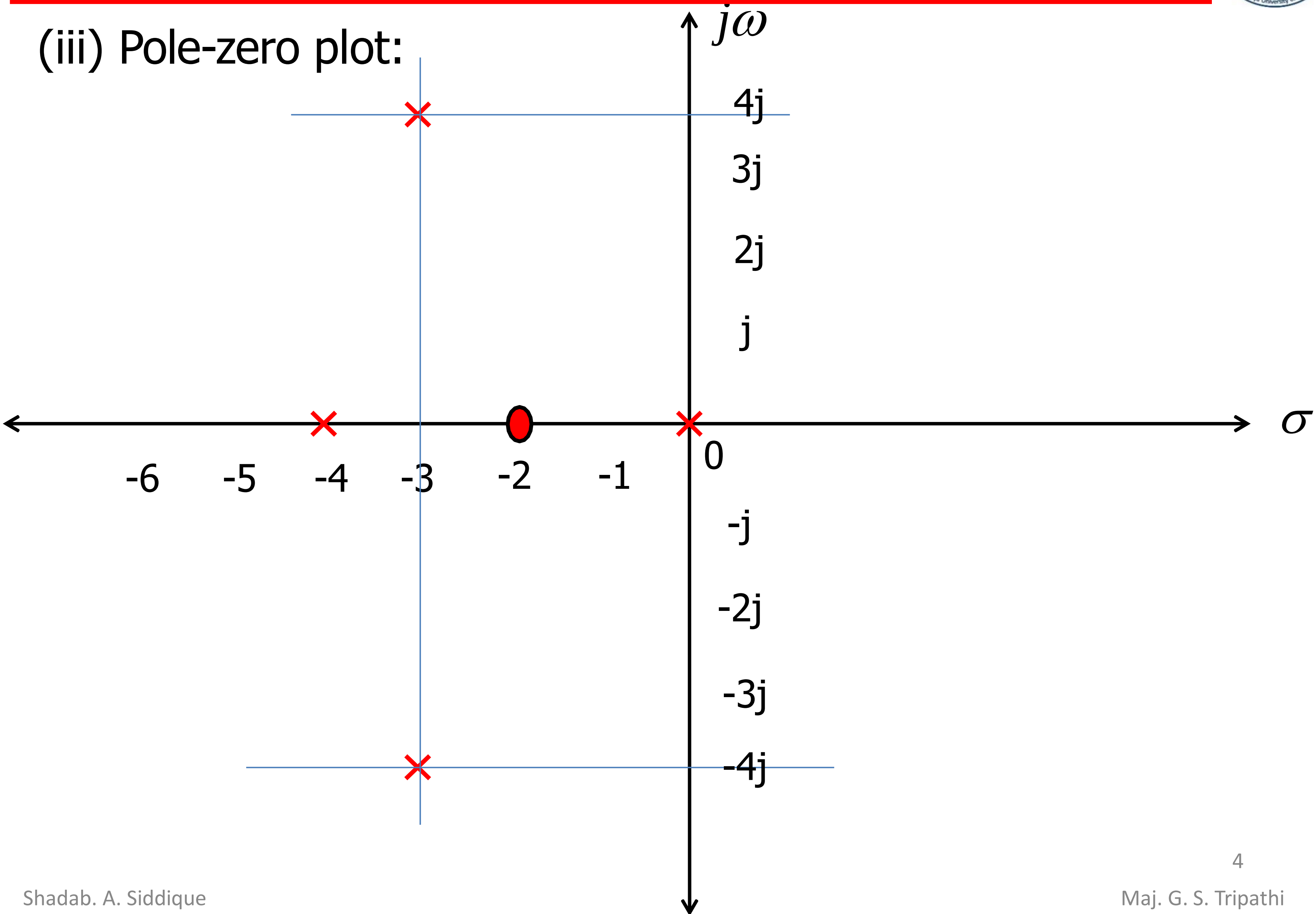
The zeros can be obtained by equating numerator with zero

$$s+2=0 \qquad \therefore s=-2$$

The zeros are s=-2

Example 2

(iii) Pole-zero plot:



(iv) Characteristics Equation:

$$s(s+4)(s^2+6s+25)=0$$

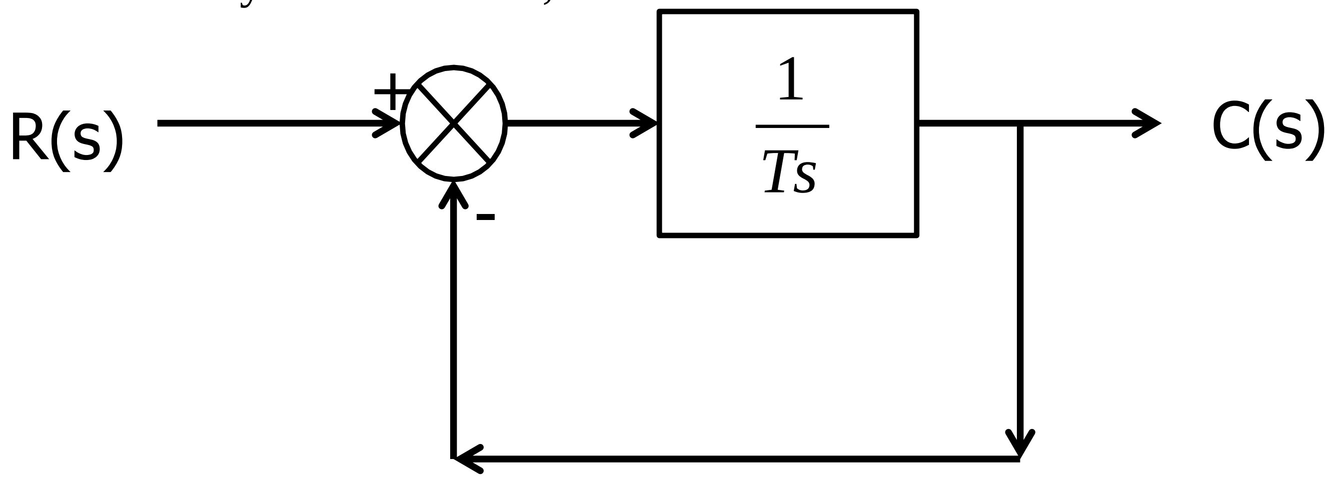
$$(s^2+4s)(s^2+6s+25)=0$$

$$\therefore s^4+6s^3+25s^2+4s^3+24s^2+100s=0$$

$$\therefore s^4+10s^3+49s^2+100s=0$$

Analysis of first order system for Step input

Consider a first order system as shown;



Here $G(s) = \dfrac{1}{Ts}$ and $H(s) = 1$

$$\therefore \frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\frac{1}{Ts}}{1 + \frac{1}{Ts}} = \frac{1}{1 + Ts}$$

Analysis of first order system for Step input

For step input;

$$r(t) = u(t) \qquad t>0$$
$$= 0 \qquad t<0$$

Taking Laplace transform;

$$R(s) = L\{Ru(t)\} = \frac{1}{s}$$

but

$$\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$

$$\therefore C(s) = \frac{1}{1+Ts} \times R(s)$$

Analysis of first order system for Step input

$$\therefore\ C(s) = \frac{1}{1+Ts} \times \frac{1}{s}$$

Using partial fraction;

$$\therefore\ C(s) = \frac{A}{s} + \frac{B}{s+\frac{1}{T}}$$

Solving;

$$\therefore\ A = s.C(s)\,|_{s=0} = 1$$

$$\therefore\ B = (s+\frac{1}{T})C(s)\,|_{s=-\frac{1}{T}} = -1$$

Analysis of first order system for Step input

$$\therefore\ C(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{T}}$$

Taking Inverse Laplace transform;

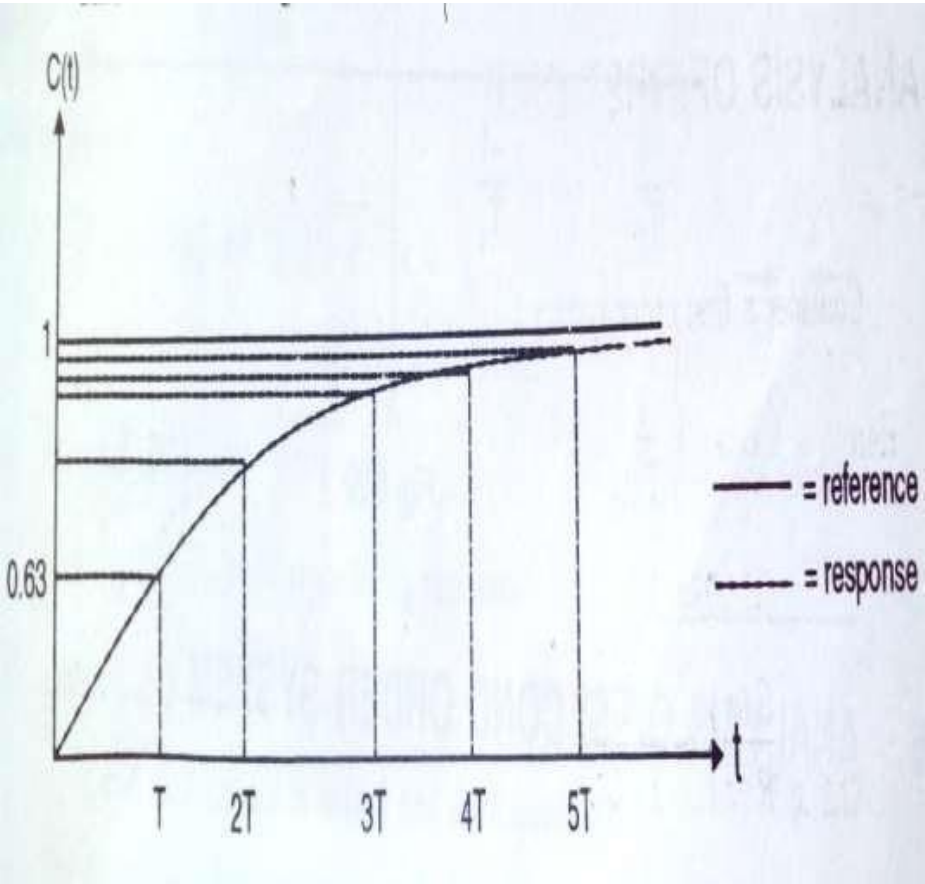
$$\therefore\ c(t) = L^{-1}\{C(s)\} = L^{-1}\{\frac{1}{s}\} - L^{-1}\{\frac{1}{s + \frac{1}{T}}\}$$

$$\therefore\ c(t) = 1 - e^{-\frac{1}{T}t}$$

Analysis of first order system for Step input

Plot c(t) vs t;

Sr. No.	t	C(t)
1	T	0.632
2	2T	0.86
3	3T	0.95
4	4T	0.982
5	5T	0.993
6	∞	1



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Time Constant (T)

- ✓ The value of c(t) = 1 only at t = ∞.
- ✓ Practically the value of c(t) is within 5% of final value at t = 3T and within 2% at t = 4T.
- ✓ In practice t = 3T or 4T may be taken as steady state.
- ✓ How quickly the value reaches steady state is a function of the time constant of the system.
- ✓ Hence smaller T indicates quicker response.

Damping

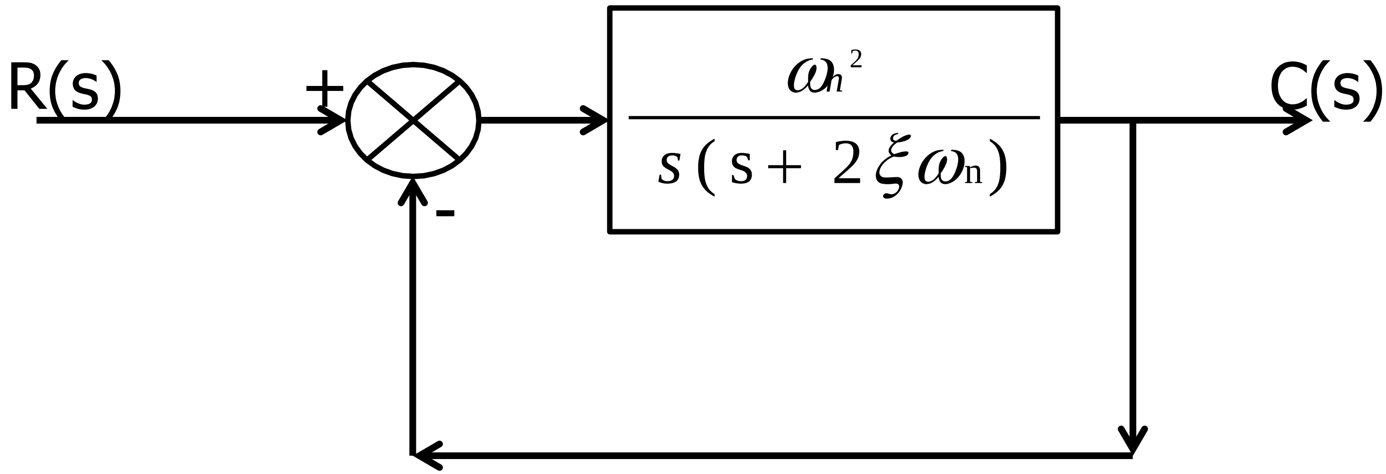
Every system has a tendency to oppose the oscillatory behavior of the system which is known as **“Damping”**.

Damping Factor ξ

The damping in any systemis measured by a factor or ratio which is known as damping ratio. It is denoted by ξ (Zeta)

Analysis of second order system for Step input

Consider a second order system as shown;



Here $G(s) = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$ and $H(s) = 1$

$$\therefore \frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Analysis of second order system for Step input

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

This is the standard form of the closed loop transfer function

These poles of transfer function are given by;

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\therefore\ s = \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4(\omega_n)^2}}{2}$$

$$= -\xi\omega_n \pm \sqrt{\xi^2\omega_n^2 - \omega_n^2}$$

$$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

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Analysis of second order system for Step input

The poles are;

(i) Real and Unequal if $\sqrt{\xi^2 - 1} > 0$

i.e. $\xi > 1$ They lie on real axis and distinct

(ii) Real and equal if $\sqrt{\xi^2 - 1} = 0$

i.e. $\xi = 1$ They are repeated on real axis

(iii) Complex if $\sqrt{\xi^2 - 1} < 0$

i.e. $\xi < 1$ Poles are in second and third quadrant