Principle of Communication (BEC-28)

Amplitude Modulation

Dr. Dharmendra Kumar
- Assistant Professor
- Department of Electronics and Communication Engineering
- MMM University of Technology, Gorakhpur–273010.
- Email: dkece@mmmut.ac.in

Correlation and Autocorrelation

- Correlation is a measure of the similarity between the waveforms.
- Correlation between $x_1(t)$ and $x_2(t)$ defined by $R_{12}(\tau)$
- $R_{12}(\tau) = T \to \infty \frac{1}{T} \int_{-T/2}^{T/2} x_1(t) x_2(t+\tau) dt$

where, $x_1(t)$ and $x_2(t)$ not necessarily periodic nor confined to finite time interval.

- If $x_1(t)$ and $x_2(t)$ are periodic with same time period T_0, then

Average cross correlation, $R_{12}(\tau) = T \to \infty \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x_1(t) x_2(t+\tau) dt$

- If $x_1(t)$ and $x_2(t)$ are finite energy signal, then

cross correlation, $R_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t) x_2(t+\tau) dt$

- Autocorrelation, when $x_1(t) = x_2(t) = x\ (t)$

Correlation and Autocorrelation….

Problem: Find $R_{12}(-1)$, $R_{12}(0)$ and $R_{21}(1)$ for signals given below

$x_1(t) = u(t) - u(t-5)$ and

$x_2(t) = 2t\big(u(t) - u(t-3)\big)$

Solution: $x_1(t)$ is nonzero for $0 \leq t \leq 5$, and $x_2(t)$ is nonzero for $0 \leq t \leq 3$

$$R_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t)x_2(t+\tau)dt$$

$$R_{12}(0) = 9$$

$$R_{12}(1) = 8 = R_{12}(-1)$$

Fourier Transform

- A periodic waveform of finite amplitude and finite frequency $f_0 = \frac{1}{T_0}$ can be expressed as sum of spectral components.
- Normalized power of above signal is also finite.
- When $T_0 \to \infty$: Above signal will be single pulse nonperiodic waveform.
- As $T_0 \to \infty$, spacing between spectral components becomes infinitesimal.
- Frequency of spectral components in Fourier series was discontinuous variable, but now it becomes a continuous variable.
- Now energy of nonperiodic signal remains finite, but power becomes infinitesimal.
- So, spectral amplitude become infinitesimal.

Fourier Transform……

- Fourier series of periodic waveform:

$$v(t) = \sum_{n=-\infty}^{\infty} V_n e^{j2\pi f_0 t}$$

- Above expression becomes

$$v(t) = \int_{-\infty}^{\infty} V(f) e^{j2\pi f t} df$$

- Finite spectral amplitudes V_n analogous to Infinitesimal spectral amplitudes $V(f)\mathrm{df}$. $V(f)$ is amplitude spectral density known as Fourier Transform of $v(t)$.
- Fourier Transform of $v(t)$:

$$V(f) = \int_{-\infty}^{\infty} v(t) e^{-j2\pi f t} dt$$

- In correspondence with V_n

$$V_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} v(t) e^{-j2\pi n f_0 t} dt$$

Fourier Transform……

- Let $v(t)$ is passed through LTI system of transfer function $H(f)$, its output:

$$v_0(t) = \int_{-\infty}^{\infty} H(f)V(f)e^{j2\pi ft}df$$

$$v_0(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} V(\omega)e^{j\omega t}d\omega$$

Problem: Find Fourier Transform of $x(t) = cos(\omega_0 t)$.

Solution: $v(t) = \frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}$; $\omega_0 = \frac{2\pi}{T_0}$

$$V(f) = \int_{-\infty}^{\infty} cos(\omega_0 t)e^{-j2\pi ft}dt$$

$$= \int_{-\infty}^{\infty} \left(\frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right)e^{-j2\pi ft}dt$$

$$= \frac{1}{2}\left(\delta(f - f_0) + \delta(f + f_0)\right)$$

Problem: Find the Fourier Transform $x(t) = \delta(t)$, a unit impulse function.

Problem: Transfer function of a network is given by $H(f)$. A unit impulse $\delta(t)$ is applied at input . Show that response at the output is the inverse transform of $H(f)$.

Fourier Transform Properties

- $v(t) \leftrightarrow V(f)$
- Time Shifting

  $v(t+\tau) \leftrightarrow V(f)e^{j2\pi f\tau}$
- Time Inversion

  $v(-t) \leftrightarrow V(-f)$
- Time Scaling

  $v(at) \leftrightarrow \frac{1}{|a|}V\left(\frac{f}{a}\right)$
- Differentiation property

  $\frac{dv}{dt} \leftrightarrow j2\pi f V(f)$
- Integration Property

  $\int_0^t v(\tau)d\tau \leftrightarrow \frac{1}{j2\pi f}V(f) + \pi V(0)\delta(f)$

  where $V(0) = \int_{-\infty}^{\infty} v(\tau)d\tau$ i.e. area under $v(t)$.

- Frequency Shifting

  $v(t)e^{i2\pi f_c t} \leftrightarrow V(f - f_c)$
- Derivative with frequency

  $-j2\pi t.v(t) \leftrightarrow \frac{dV(f)}{df}$
- Duality or Symmetry

  $V(t) \leftrightarrow v(-f)$

  $V(t) \leftrightarrow 2\pi v(-\omega)$
- Linearity

  $av_1(t) + bv_2(t) \leftrightarrow aV_1(f) + bV_2(f)$

  where $v_1(t) \leftrightarrow V_1(f)$ and $v_2(t) \leftrightarrow V_2(f)$

Examples

Problem: Find Fourier Transform of the signal $v(t) = e^{-at}u(t)$ where $u(t)$ is unit step function.

Solution: $V(f) = \int_{-\infty}^{\infty} u(t)e^{-at}e^{-j2\pi ft}dt$

$$= \int_{0}^{\infty} e^{-(a+j2\pi f)t}dt = \frac{1}{a+j2\pi f}$$

Problem: Find Fourier Transform of the signal $v(t) = e^{-a|t|}$ where a is positive real number.

Solution: $e^{-a|t|} = e^{-at}u(t) + e^{at}u(-t)$

$$e^{-at}u(t) \leftrightarrow \frac{1}{a+j2\pi f}$$

Time Reversal property, $e^{at}u(-t) \leftrightarrow \frac{1}{a+j2\pi(-f)}$

Linearity property, $e^{-a|t|} \leftrightarrow \frac{1}{a+j2\pi f} + \frac{1}{a+j2\pi(-f)}$

$$e^{-a|t|} \leftrightarrow \frac{2a}{a^2+4\pi^2 f^2}$$

Problem: Find Fourier transform of $\text{sgn}(t) = u(t) - u(-t)$.

Solution: $\text{sgn}(t) = a \to 0[e^{-at}u(t) - e^{at}u(-t)]$

Using linearity property, $\text{sgn}(t) \leftrightarrow \frac{1}{j\pi f}$

Problem: Find inverse Fourier transform of $-jsgn(\omega)$.

Solution: From the above problem, $sgn(\omega) = \frac{2}{j\omega}$

$$IFT[-jsgn(\omega)] = jIFT[sgn(-\omega)]$$

Duality property, $IFT[-jsgn(\omega)] = \frac{1}{\pi t}$

Thank You